Biological and Environmental Physics

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8 Lectures:

- Exponential Growth and Decay
- Energy Balance of the Earth
- Physics of the Atmosphere
- Health Physics

These notes can be found at:

http://www.physics.gla.ac.uk/~dmiller (click on ‘teaching’)

1. Exponential Growth and Decay

Rate of change of a quantity is proportional to quantity's size $\longrightarrow$ Exponential Growth

"Change" could be with time, e.g. population of bacteria, or may be something else, e.g. distance

Example: population of cells in a culture

Growth rate: 10% per day

$100 \Rightarrow 110$	after 1st day	i.e. 100 + 10
$110 \Rightarrow 121$	after 2nd day	i.e. 110 + 11
$121 \Rightarrow 133.1$	after 3rd day	i.e. 121 + 12.1

N = number of cells, dN is the change in number after a time dt (here 1 day)

$$dN = 0.1 \times N \times dt$$

If dt is *small*, then:

Rate of change of number = rate of increase $\times$ Number

growth constant

$$\boxed{\frac{dN}{dt} = \lambda N}$$

$$\frac{dN}{dt} = \lambda N$$

$$\int \frac{1}{N} dN = \int \lambda \, dt$$

$$log_e N = \lambda t + \text{Constant}$$

$$log_e N_0$$

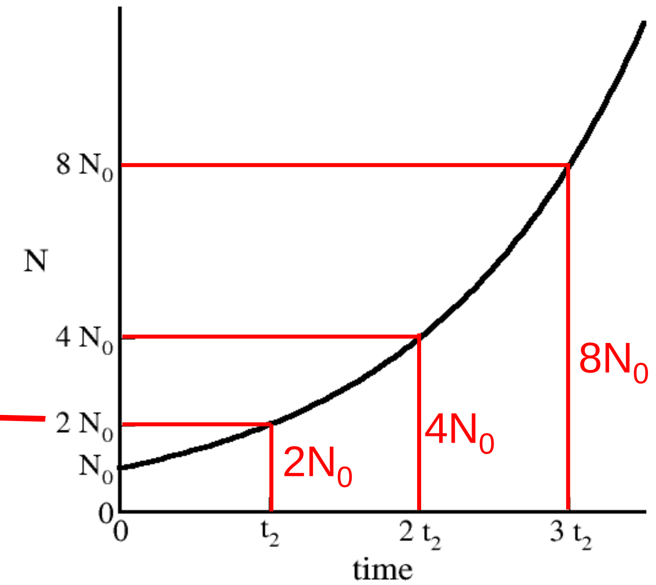
$$\boxed{N = N_0 e^{\lambda t}}$$

Note when t = 0, N = N_0 (recall e = 2.718 and $e^0 = 1$)

Doubling time

How long does it take the population to double?

Let the ***doubling time*** be t_2 , so $N = 2N_0$ at $t = t_2$



Calculating the doubling time:

$$2N_0 = N_0\, e^{\lambda\, t2}$$

Cancel N_0 and take logs (base e) of both sides:

$$\lambda\, t_2 = \log_e 2 = 0.693$$

In our previous example, $\lambda = 0.1$, so $t_2 = \log_e 2 / \lambda = 0.693 / 0.1 = 6.93$ days

[But note that at t = 3 days, $N = 100\, e^{0.1 \times 3} = 135$. This is not what we saw earlier because dt = 1 day is ***not*** small compared with the doubling time.]

The Radioactive Decay Law

The rate of radioactive decay is proportional to the number ***N*** of nuclei present.

Rate of ***increase*** of number = **-** λ N (this is a decrease, since sign is **-**)

λ is the decay constant: the probability that a nucleus decays in unit time

$$\frac{dN}{dt} = -\lambda N$$

When t = 0, N = N_0, so

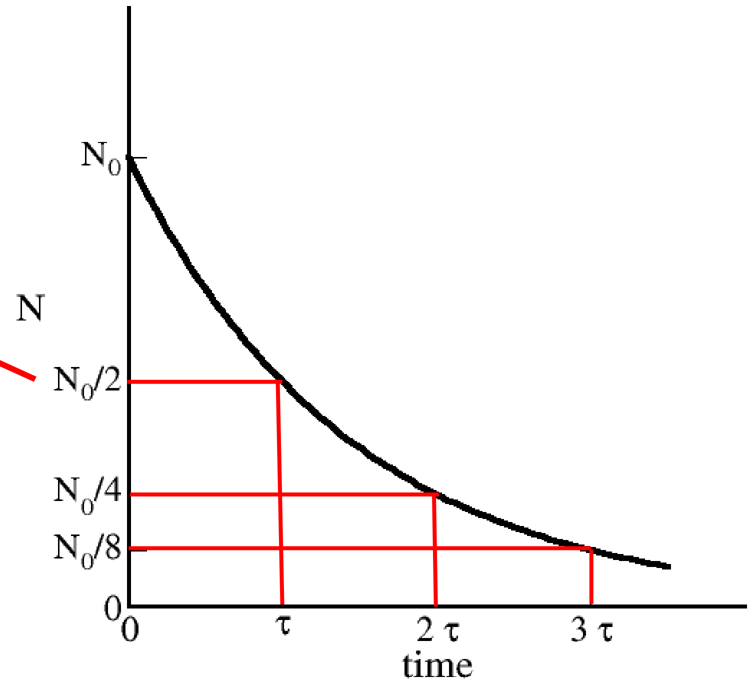
$$\boxed{N = N_0 e^{-\lambda t}}$$

Half-Life

Half-life is the time taken for ***half*** of the nuclei in the sample to decay

If at $t = 0, N = N_0$ then at $t = \tau , N = N_0 / 2$

$$N_0 / 2 = N_0 e^{-\lambda \tau}$$



Cancel N_0 and take logs (base e) of both sides:

$$\log_e ½ = - \lambda \tau$$

$$\log_e 1 - \log_e 2 = - \lambda \tau$$

$$\boxed{\lambda \tau = \log_e 2 = 0.693}$$

(Just as for doubling time)

An Example

A sample of U-238 has a disintegration rate of 250 disintegrations per minute.
If the half-life of U-238 is 4.51×10^9 years, what mass of U-238 is present in the sample?

Know the half-life τ and need to work out λ for the decay law:

$$\lambda\tau = \log_e 2 \Rightarrow \lambda = \frac{\log_e 2}{\tau} = \frac{0.693}{4.51\times 10^9 \mathrm{y}} = 1.537 \times 10^{-10} \mathrm{y}^{-1}$$

Note: this is years

Number of disintegrations in one year = $250 \times$ number of minutes in a year
= $250 \times (60 \times 24 \times 365)$
= 1.314×10^8

Since dt (1 year) is *small* compared with τ , we can use $\frac{dN}{dt} = -\lambda N$

Rate of increase of N is $\frac{dN}{dt} = -1.314 \times 10^8$ nuclei / year

$$N = -\frac{1}{\lambda}\frac{dN}{dt} = -\frac{-1.314\times 10^8 \text{ nuclei y}^{-1}}{1.537\times 10^{-10}\mathrm{y}^{-1}} = \underline{8.549 \times 10^{17} \text{ nuclei}}$$

Now calculate the mass…

Growth and decline of populations

This is also an exponential process.
Growth of a population depends on the number of births and thus the size of the population.
Decline of a population depends on the number of deaths and thus the size of the population.

Must factor in both births and deaths at once.

Example

Each year a population has 30 births and 20 deaths per 1000 members of the population. How many years will it take the population to double?

Net rate of increase = (30 – 20) / 1000 per person per year

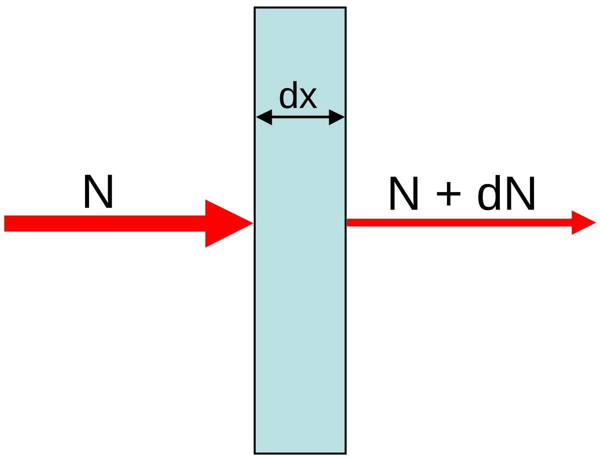
$$\Rightarrow \lambda = 0.01 \text{ y}^{-1}$$

Doubling time, $t_2 = \log_e 2 / \lambda = 0.693 / 0.01\text{y}^{-1} = 69.3$ years

[In practice, birth and death rates will depend on more than just population size!]

Absorption processes

Imagine light (or X-rays, nuclear radiation etc.) passing through a material (glass, perspex, air etc.). The number of photons which are absorbed, ***dN***, depends on the original number of photons, ***N***, and the distance travelled, ***dx***.



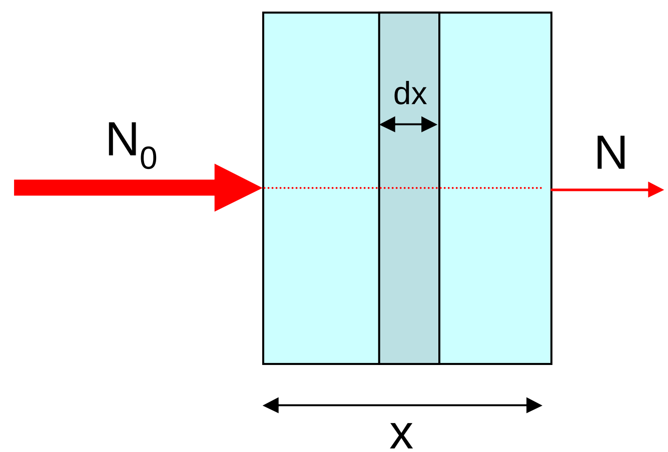
$$dN = -\mu N dx$$

absorption coefficient

$$\frac{dN}{dx} = -\mu N$$

If dx is small:

$$\boxed{N = N_0 e^{-\mu x}}$$



An Example

It is found that lead sheet of thickness 22mm attenuates the gamma radiation from Cs-137 by a factor of 10. What is the linear absorption coefficient μ in this case? What thickness of lead will attenuate by a factor of 200?

Intensity: $I = I_0 e^{-\mu x}$ When $x = 22\text{mm},\ I = \frac{1}{10} I_0$

Rearrange:

$$\log_e\left(\frac{I}{I_0}\right) = -\mu x \qquad \Rightarrow \qquad \mu = -\frac{1}{x}\log_e\left(\frac{I}{I_0}\right)$$

Put in some numbers:

$$\mu = -\frac{1}{22\,\text{mm}}\log_e\left(\frac{1}{10}\right) = \frac{1}{22\,\text{mm}}\log_e 10 = \underline{0.105\ \text{mm}^{-1}}$$

Now, if attenuation is a factor of 200:

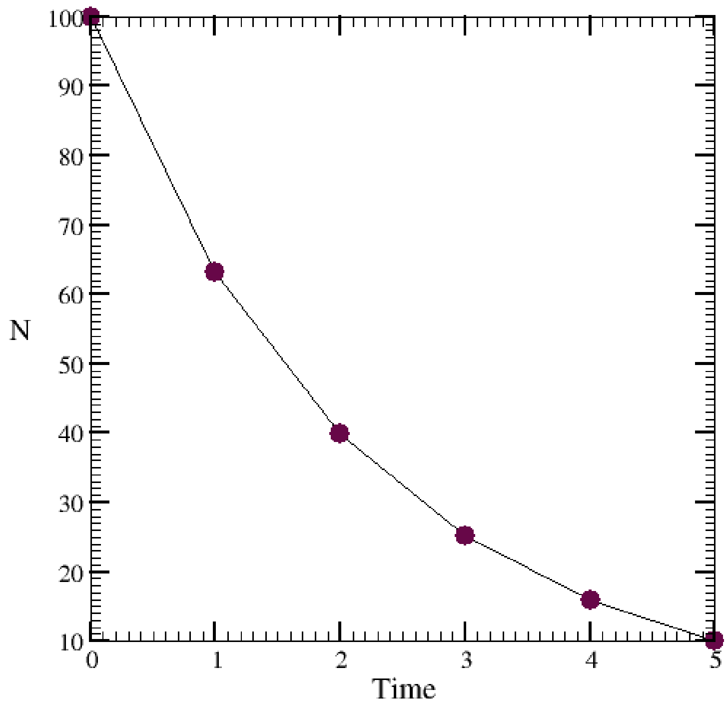
$$x = -\frac{1}{\mu}\log_e\left(\frac{I}{I_0}\right) = -\frac{1}{0.105\,\text{mm}^{-1}}\log_e\frac{1}{200} = \underline{50.5\ \text{mm}}$$

Instead of ***half-life***, we have ***half-distance***.
Here the half distance is 6.6mm. Can you show this?

Log–linear graph paper

Consider the following data:

Time (s)	0	1	2	3	4	5
N (counts/s)	100	63.1	39.81	25.12	15.85	10.0
Log_{10} N	2.00	1.8	1.6	1.4	1.2	1.0



This is an exponential decay with $\lambda_{10} = 0.2\ \text{s}^{-1}$

$$N = N_0\, 10^{-\lambda_{10} t}$$

[it doesn't matter that this is $10^{-\lambda_{10}t}$ rather than $e^{-\lambda_e t}$, since $10^x = e^{x \log_e 10}$.
So $N = N_0\, e^{-\lambda_{10}\, t \log_e 10}$ and $\lambda_e = \lambda_{10} \log_e 10 = 0.46\ \text{s}^{-1}$]

But the relation is easier to work with in terms of logs

$$\log_{10} N = \log_{10}\left(N_0\, 10^{-\lambda t}\right) = \log_{10} N_0 - \lambda t$$

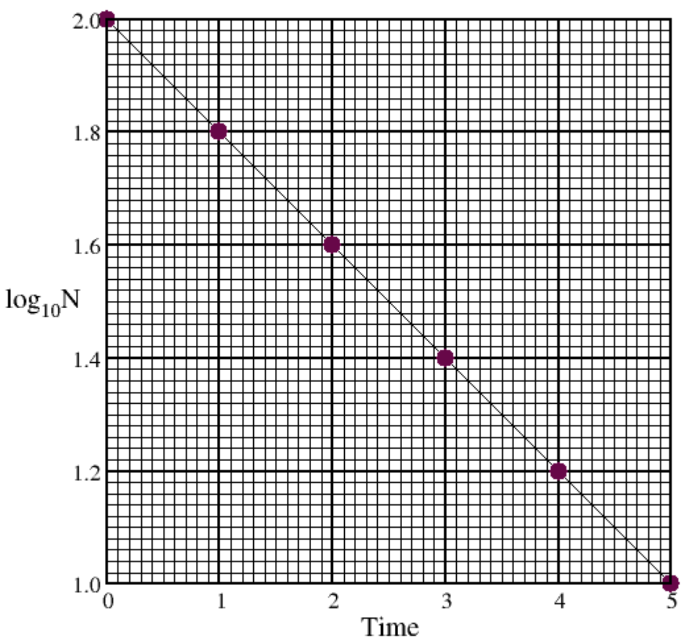
Change in *$\log_{10} N$* is linear in t

$$\log_{10} N = \log_{10} N_0 - \lambda t$$

Normal graph paper

Graph of ***$log_{10}N$*** against t is ***straight line*** with starting value $\log_{10}N_0$ and gradient $-\lambda$ and

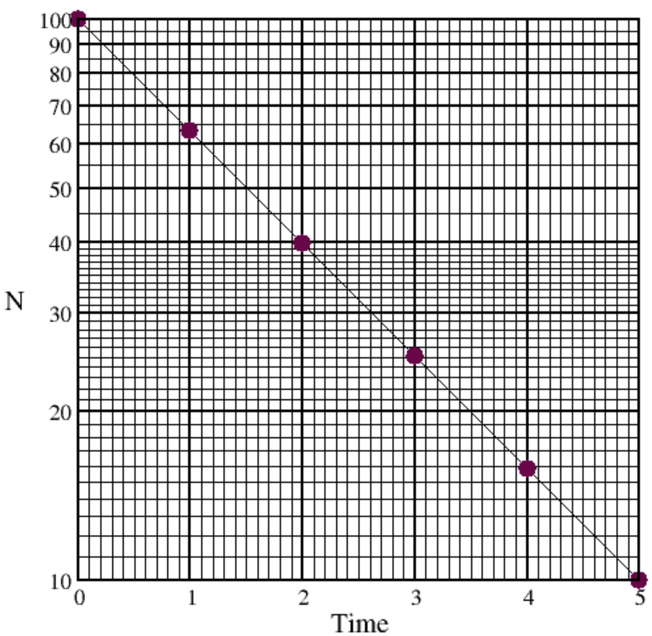
Gradient = - (2.0 -1.0) / (5 – 0) = -0.2 s^{-1}



Log-linear graph paper

Now plot on log-linear graph paper.
Points and joining line look ***identical*** to before.

Gradient = - ($\log_{10}100 - \log_{10}10$) / (5 – 0) = -0.2 s^{-1}



Half-life is $\tau = \log_e 2 / \lambda_e = \log_e 2 / (0.2\ s^{-1} \times \log_e 10) = 1.51$ s

Summary of Exponential Growth and Decay

If the rate of change of a sample size is proportional to the sample size,
we have exponential growth or decay.

$$\frac{dN}{dt} = \lambda N \qquad\qquad N = N_0 e^{\lambda t}$$

λ = decay/growth constant

The doubling time, t_2 is the time taken for a sample to double: $\lambda\, t_2 = \log_e 2$

The half-life, τ, is the time taken for a sample to halve: $\lambda\, \tau = \log_e 2$

Absorption is an exponential process with time replaced by distance
The half-distance is the distance required to absorb half the incoming light.

Exponential processes become linear when one uses a logarithmic scale
log-linear graph paper can be useful for plotting exponential processes in this way

2. Energy Balance of the Earth

Energy from the Sun

The sun is powered by fusion reactions. Isotopes of hydrogen collide in the sun and fuse together, finally forming helium.

Overall reaction: $4\ {}^{1}_{1}H + 2\ e^{-} \longrightarrow {}^{4}_{2}He + 2\ \nu + 6\ \gamma$

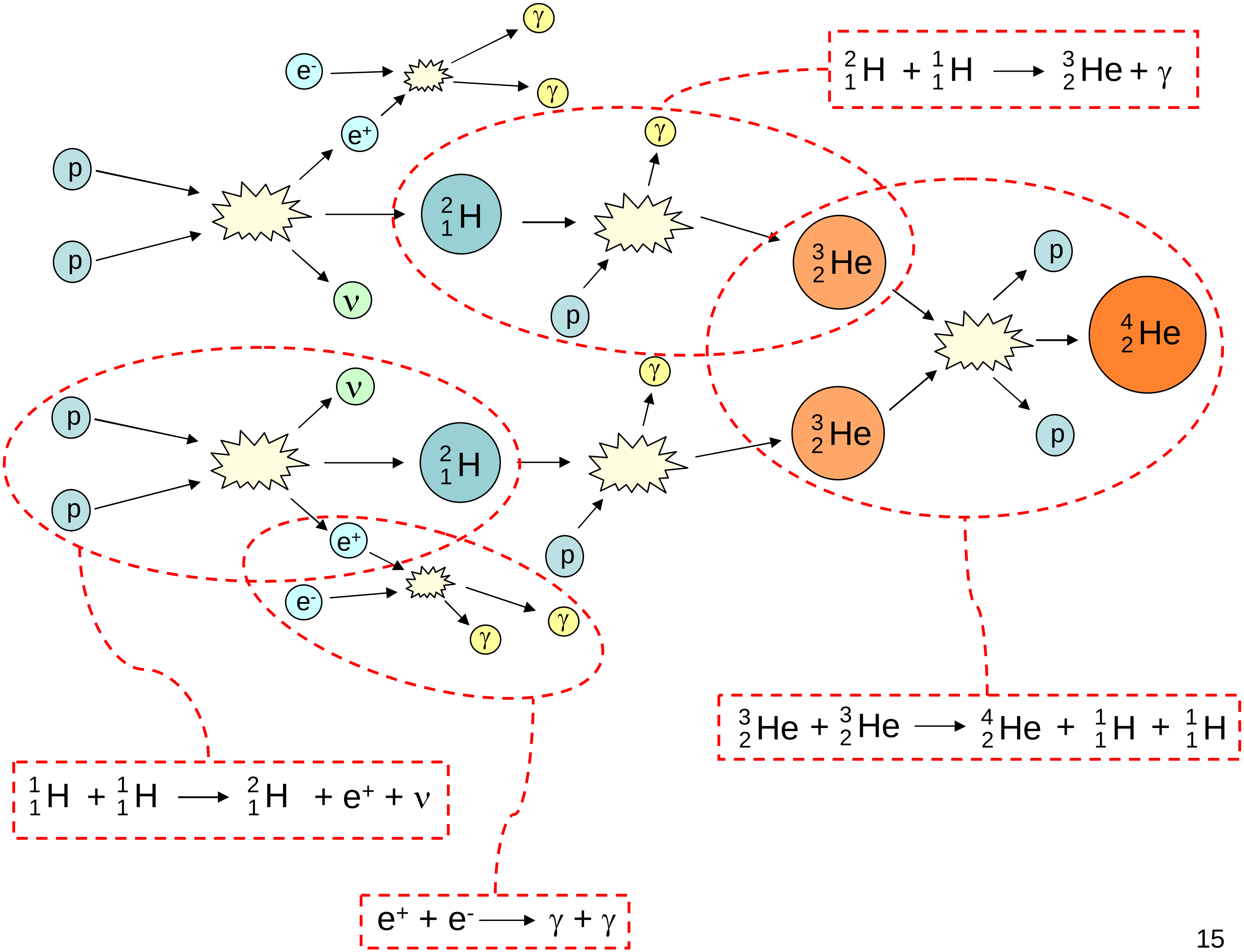
proton, electron, alpha radiation, neutrino, photon

The mass of the final state is lower than the initial state.
$E=mc^2$ tells us that the mass is converted into energy:

energy output = 26.7 MeV = 4.28×10^{-12} J

Protons are charged, so to get them to collide we have to overcome the Coulomb repulsion.
In the sun, very high temperatures (15×10^{6} K) mean the protons have high energy and may fuse.

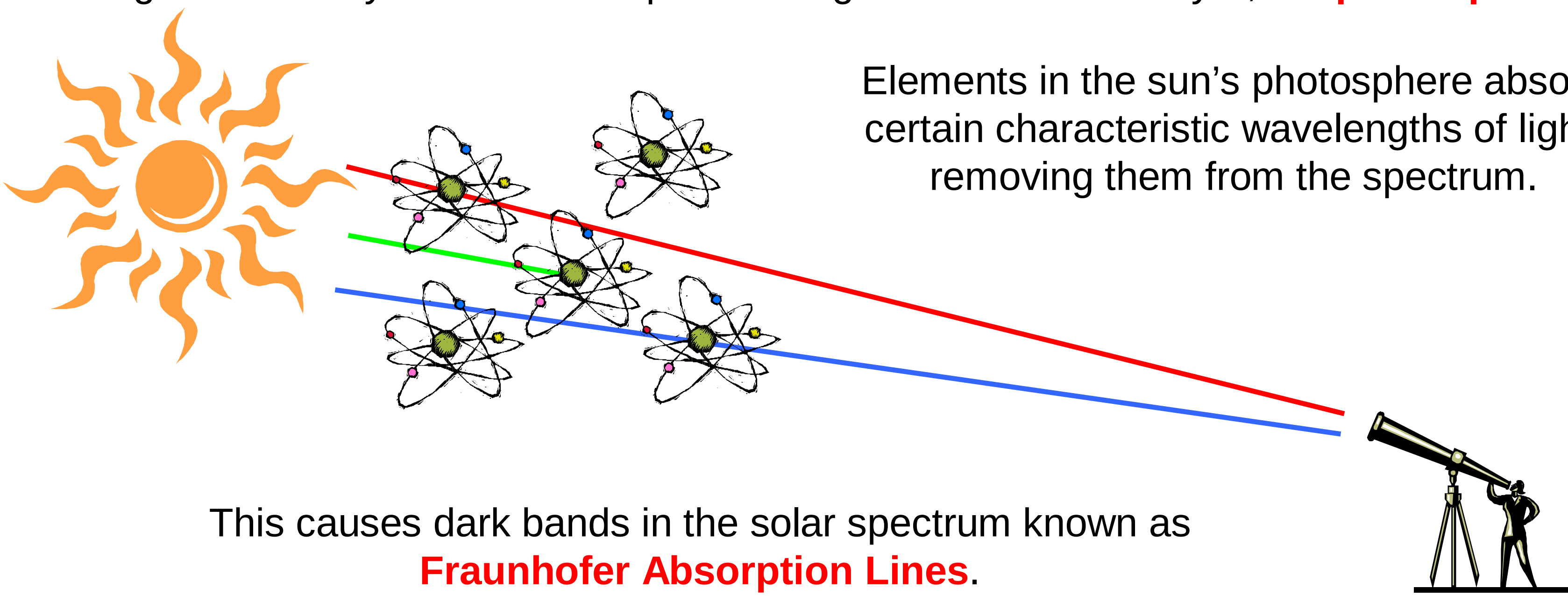
Hence we call this is ***thermonuclear fusion***.



$^{2}_{1}H + ^{1}_{1}H \longrightarrow ^{3}_{2}He + \gamma$
$^{1}_{1}H + ^{1}_{1}H \longrightarrow ^{2}_{1}H + e^{+} + \nu$
$e^{+} + e^{-} \longrightarrow \gamma + \gamma$
$^{3}_{2}He + ^{3}_{2}He \longrightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$
p
e-
e+
γ
ν
$^{2}_{1}H$
$^{3}_{2}He$
$^{4}_{2}He$

Fraunhofer Absorption Lines

In 1814 Joseph von Fraunhofer, an optician, observed dark lines in the solar spectrum. Light emitted by the sun much pass through the sun’s outer layer, the **photosphere**.



Elements in the sun’s photosphere absorb certain characteristic wavelengths of light, removing them from the spectrum.

This causes dark bands in the solar spectrum known as **Fraunhofer Absorption Lines**.

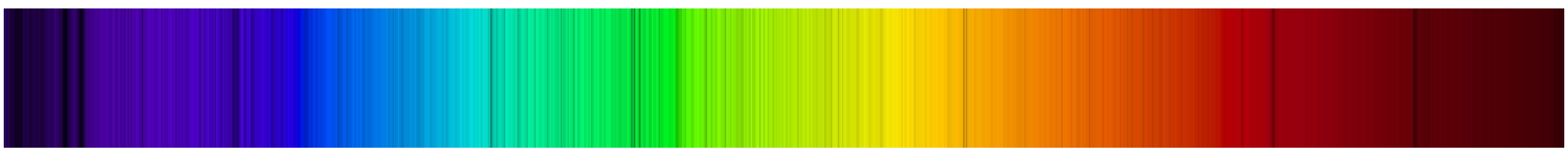


Image courtesy Mees Solar Observatory

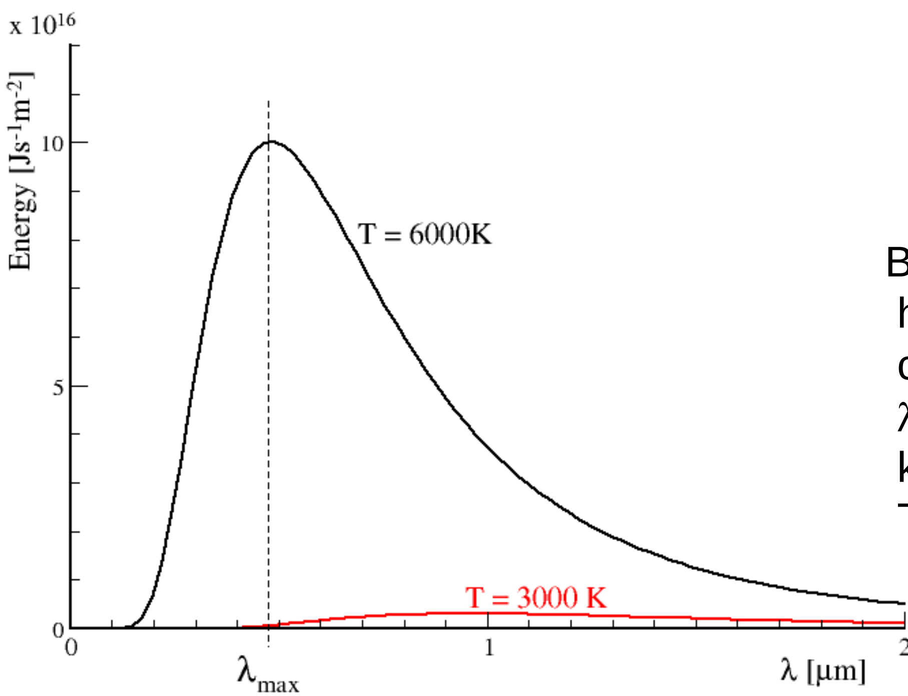
Although Fraunhofer mapped only 600 or so lines, we now know of more than 3,000!

Caused by: H, He, Mg, Ca, Fe (and many others) in the photosphere
O_2 in the Earth’s atmosphere

Black body radiation

A **black body** is an object which absorbs **all** radiation which falls on it.
It reflects no light and is ‘black’.

A black body emits the maximum amount of energy at every wavelength (perfect emitter)
This gives the **black body spectrum**.
Shows how much energy a black body of temperature T emits at a wavelength λ:



$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

B_λ = Energy emitted per *s* per m^2 area
h = Planck's constant (6.626 x 10^{-34} *Js*)
c = Speed of Light (3 x 108 ms^{-1})
λ = Wavelength of light (*m*)
k = Boltzmann Constant (1.38 x 10^{-23} JK^{-1})
T = Temperature (*K*)

The sun is a near perfect black body with temperature ≈ 6000 K

Stefan-Boltzmann Law

(Sometimes called Stefan's Law)

Tells us the total energy emitted per m^2 per s.

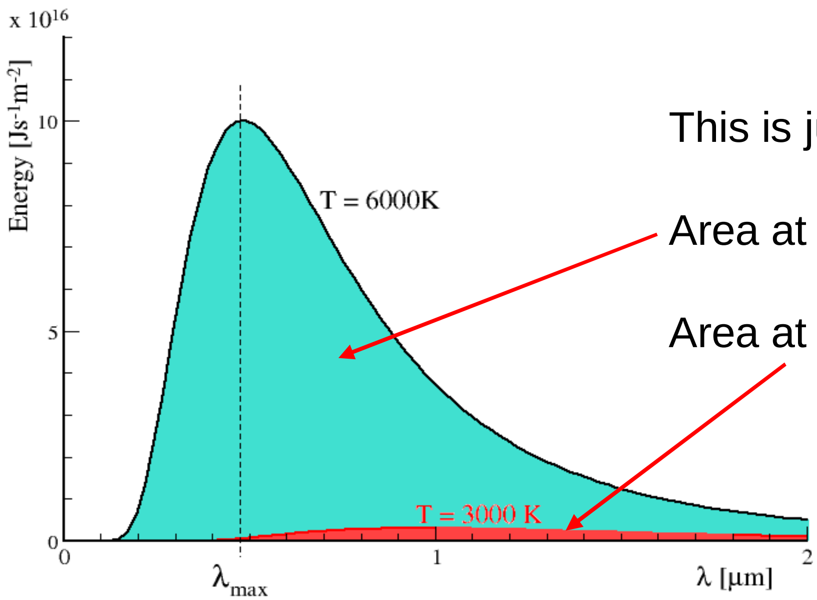
$$I = \varepsilon \sigma T^4$$

Temperature (K)

Emissivity

For a perfect black body $\varepsilon = 1$,
but not all bodies emit perfectly
If $\varepsilon < 1$ we have a grey body
For the sun, $\varepsilon \approx 1$

Stefan-Boltzmann constant
$\sigma = 5.67 \times 10^{-8}$ Js^{-1}m^{-2}K^{-4}



This is just the area under the black body curve.

Area at 6000 K ← $2^4 = 16$ times larger

Area at 3000 K

Wien's Displacement Law

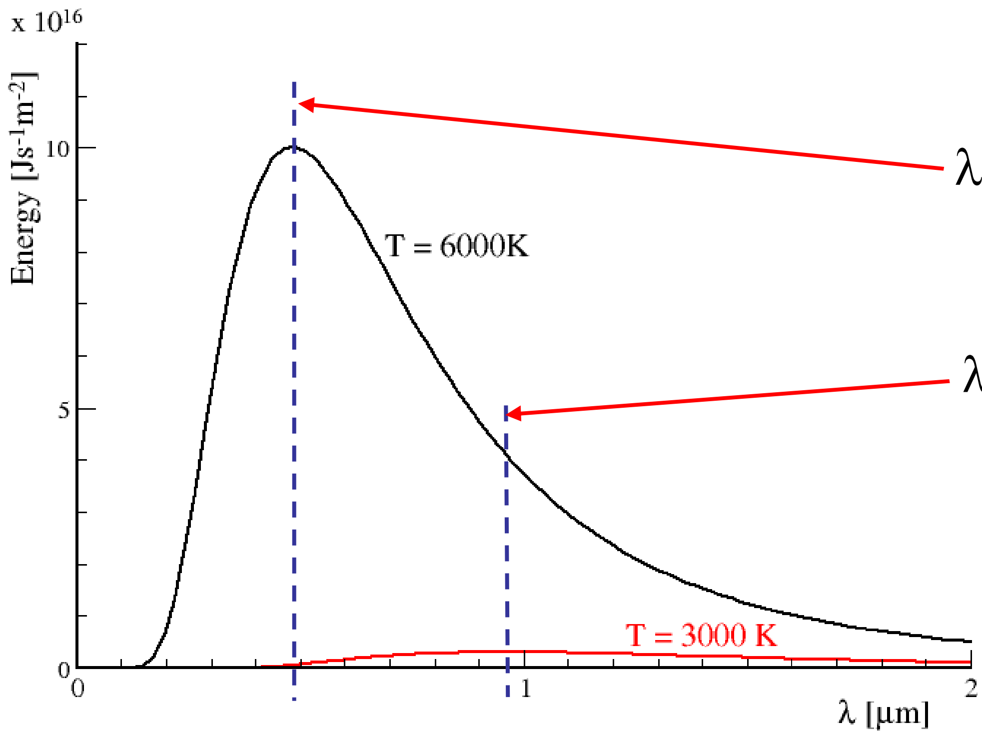
The maximum of the black body spectrum is at a wavelength

$$\boxed{\lambda_{max} = \frac{a}{T}}$$

constant
2.898×10^{-3} m K

Wavelength which is emitted most (m)

Temperature of black body (K)



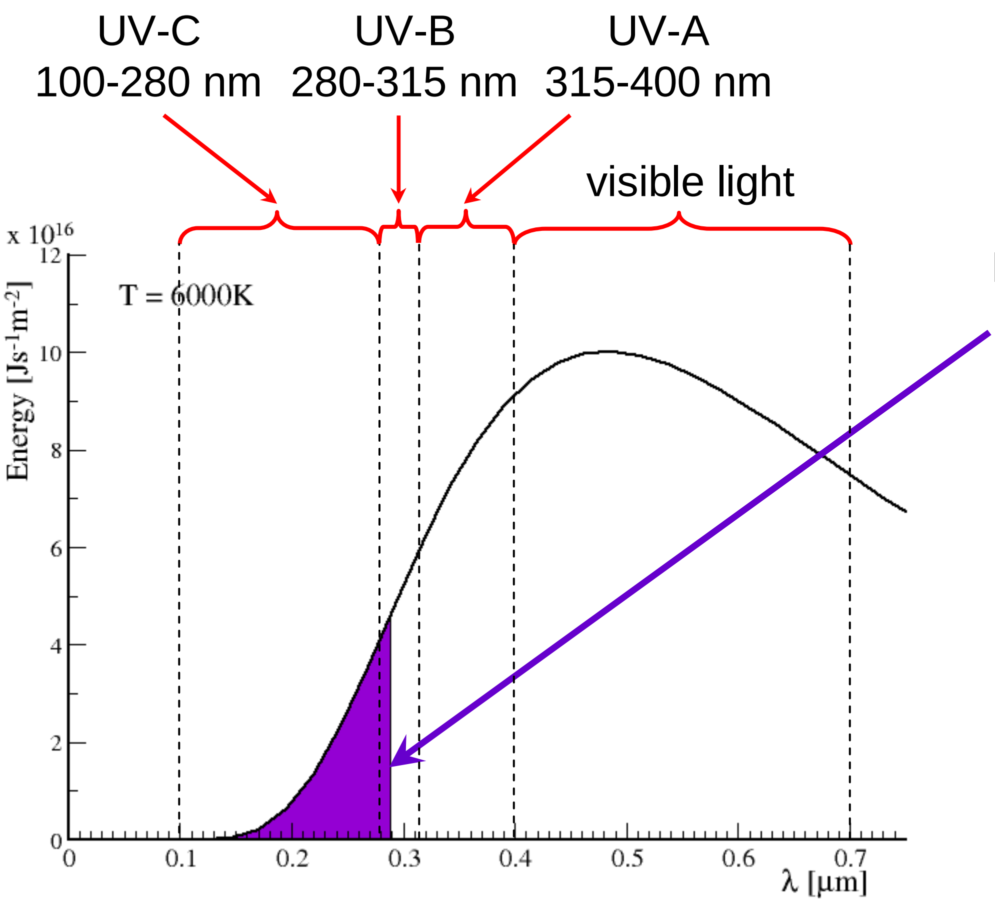
$\lambda_{max} = 2.898 \times 10^{-3}$ m K / 6000 K = 0.483 μm

$\lambda_{max} = 2.898 \times 10^{-3}$ m K / 3000 K = 0.966 μm

Lower temperature black bodies
emit less radiation
but emit at higher wavelengths

Sunlight at the Earth's Surface

Ultraviolet radiation can be damaging to life on Earth. UV-A causes little sunburn and slightly darkens skin pigmentation, UV-B can cause more serious sunburn, thickening and loss of elasticity of skin, and even skin cancers. UV-C can kill micro-organisms, damage proteins and DNA and cause keratitis and conjunctivitis (eye complaints).



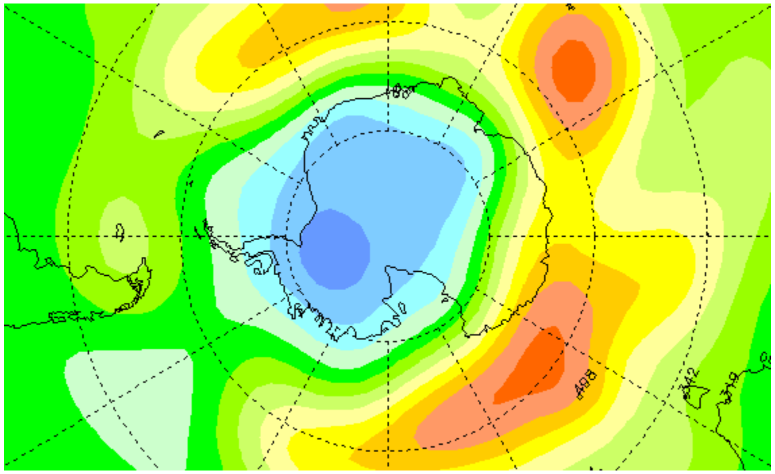
Fortunately, ozone (O_3) absorbs ultraviolet wavelengths of light with $\lambda < 290$ nm.

UV-A, B and C constitute 7% of sun's spectrum, but only 3% reaches the Earth.

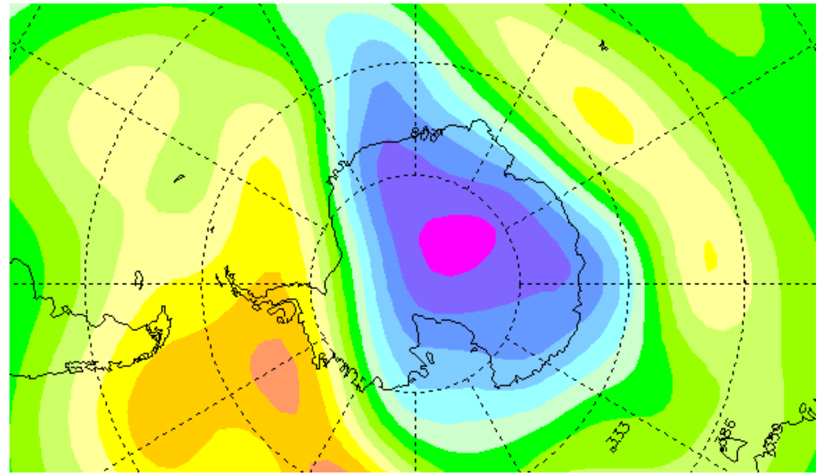
Unfortunately, an Ozone hole has recently appeared over the Antarctic.

The Ozone Hole

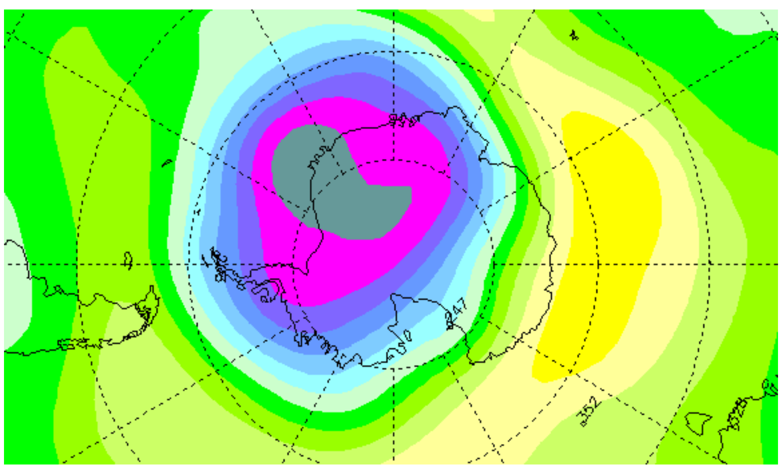
Pictures from British Antarctic Survey



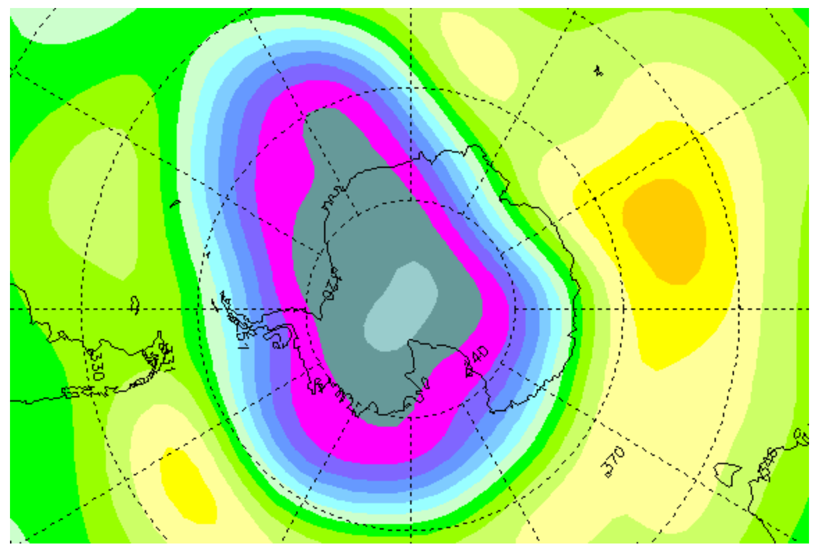
October 1980



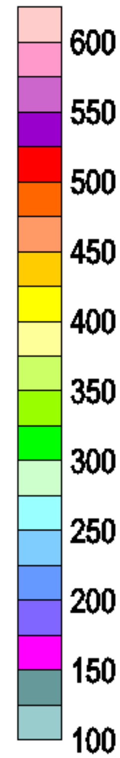
October 1986

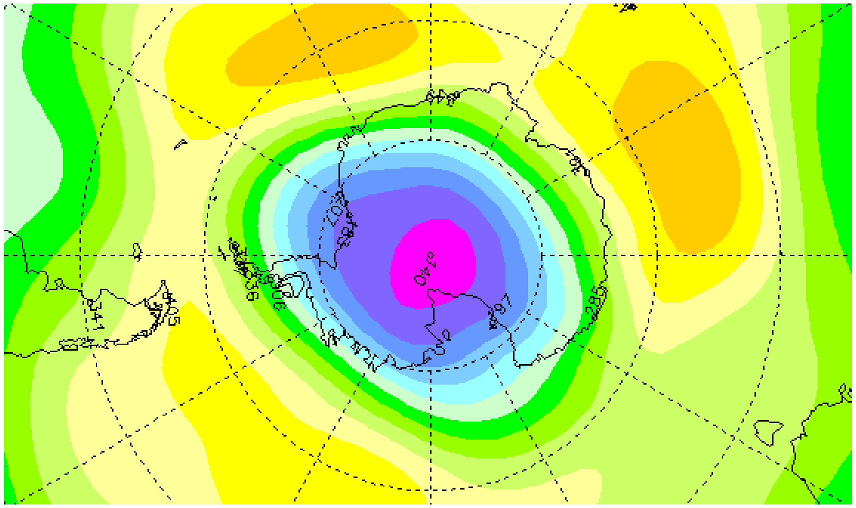


October 1992

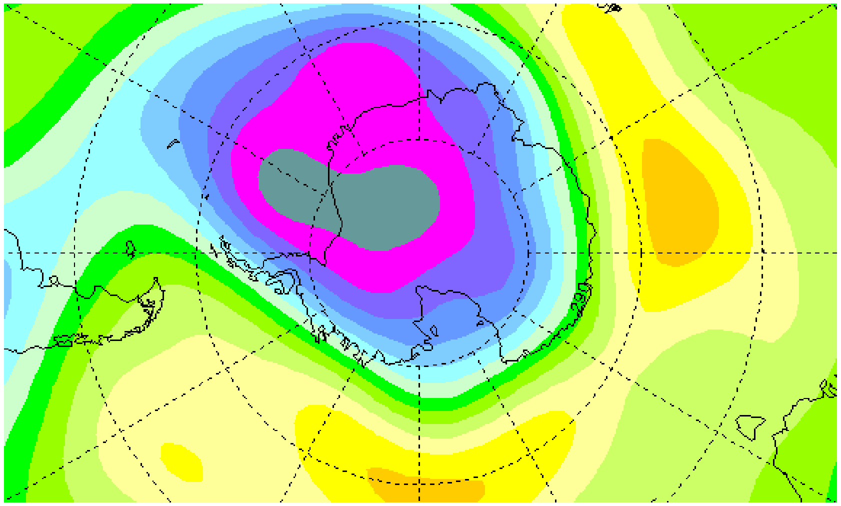


October 1998

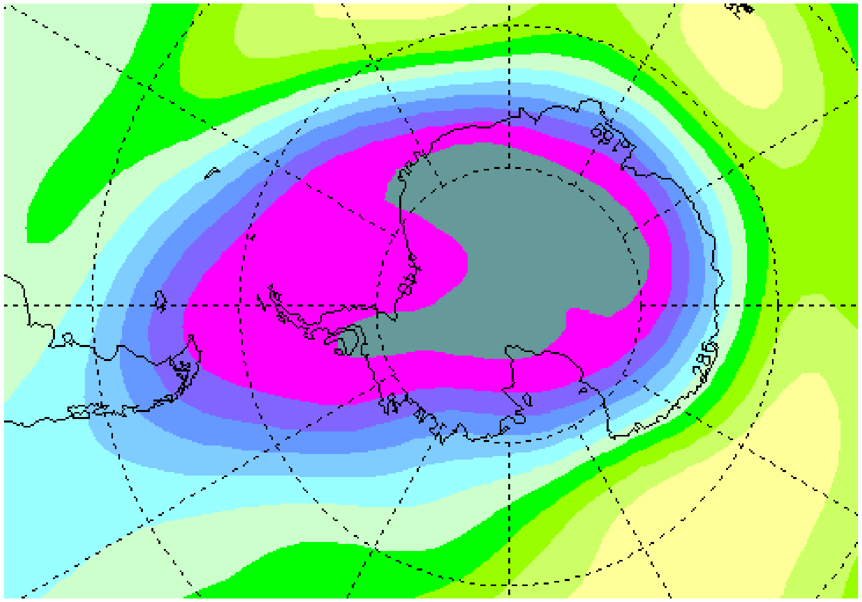




October 2002



October 2003



October 2004

In 2002 the ozone hole appeared smaller but this is thought to just be an effect of unusual weather patterns.

Once again, had a very large hole in 2004

What Causes the Ozone Hole?

The ozone layer is broken down by gasses containing chlorine and bromine atoms (halogens), such as **chlorofluorocarbon (CFC)** molecules.

These chemicals are man made and have been used inrefrigerants, anesthetics, aerosols, fire-fighting equipment and the manufacture of polystyrene.

CFC's (

) are broken down into chlorine by UV light in the atmosphere

$$CFCl_3 + \text{UV Light} \rightarrow CFCl_2 + Cl$$

Chlorine atoms then break down the ozone:

$$Cl + O_3 \rightarrow ClO + O_2$$
$$ClO + O \rightarrow Cl + O_2$$

One chlorine atom can destroy 100,000 ozone molecules!

In Antarctic regions, **Polar Stratospheric Clouds (PSC)** greatly increase the abundance of halogens

$\Rightarrow$ Antarctic Ozone hole

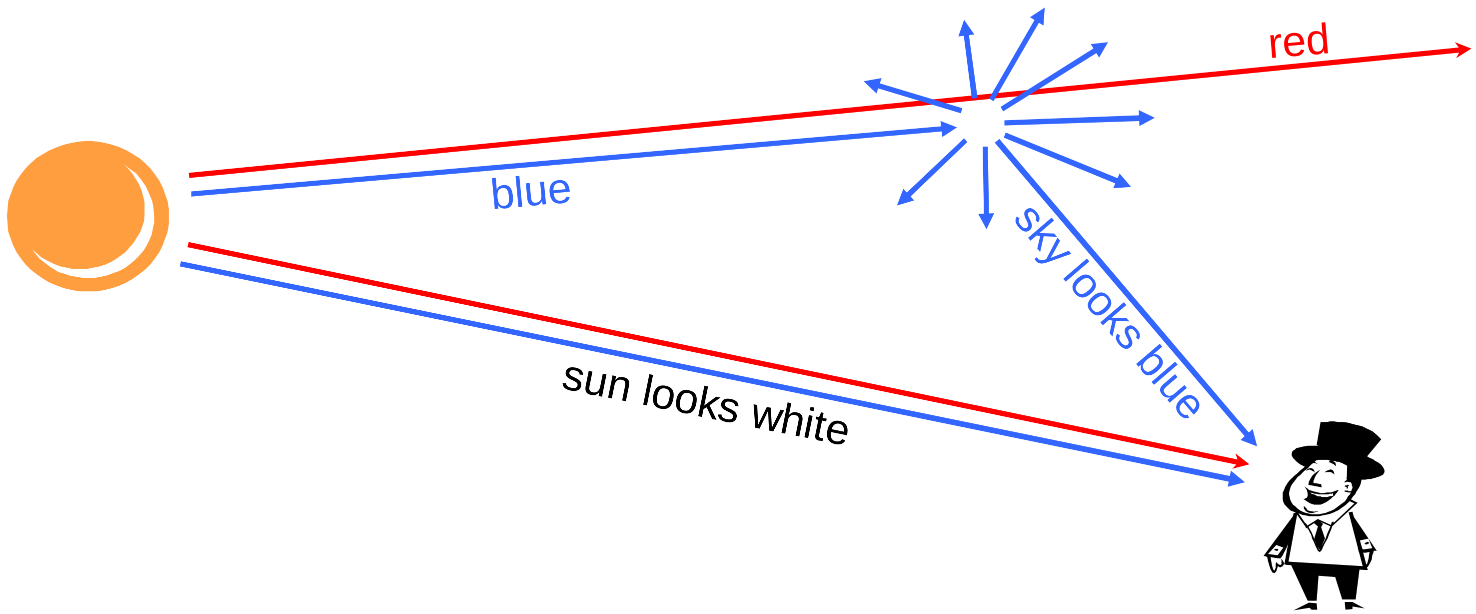
Rayleigh's Law for scattering of light

How much light is scattered depends **inversely** on the **fourth power** of its **wavelength**

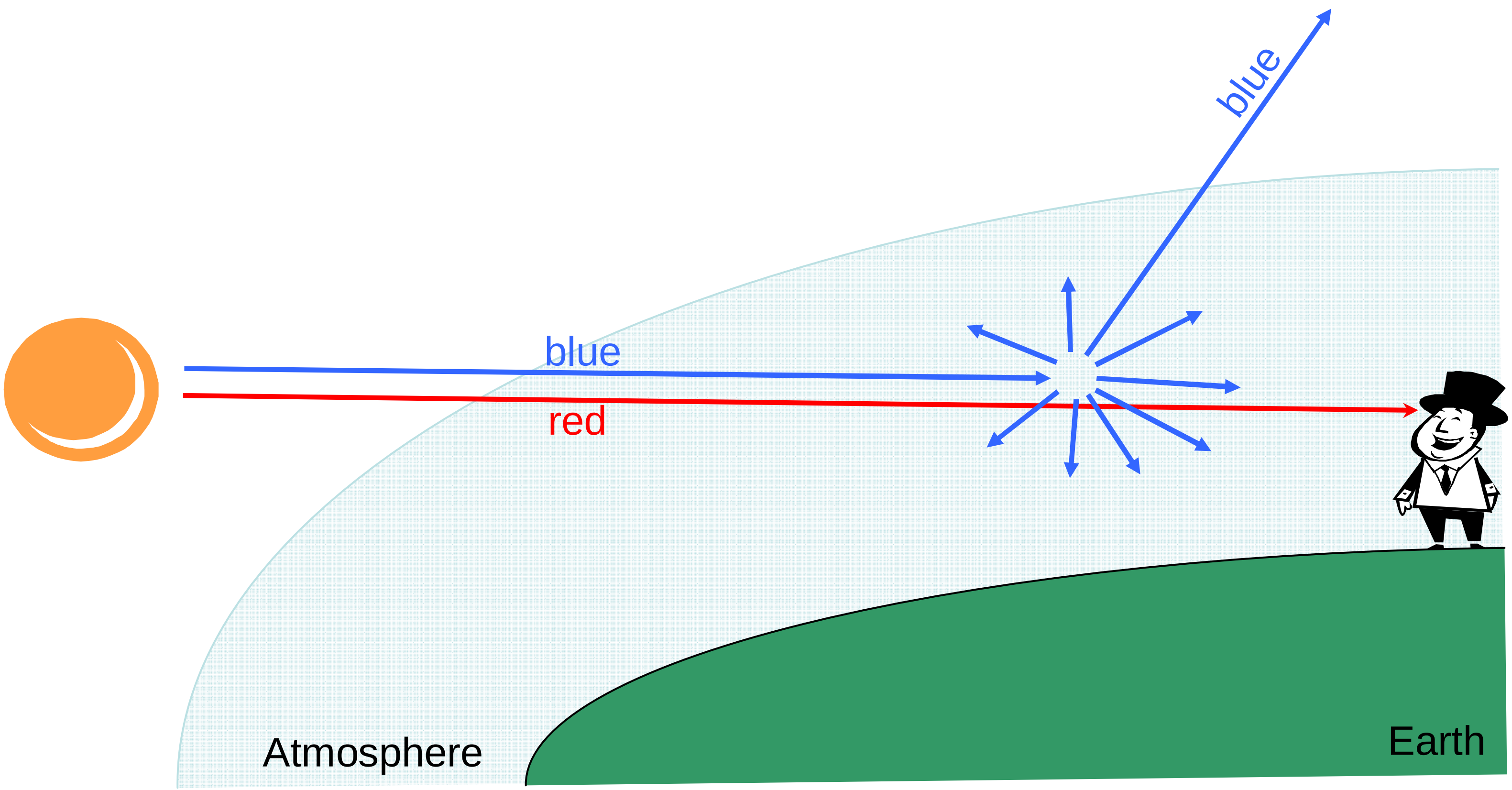
$$\textbf{Scattering} \propto \frac{1}{\lambda^4}$$

Blue light (400nm) is scattered 9 times as much as **red light** (700nm).

$\Rightarrow$ Sky is blue because blue is scattered more



⇒ Sunsets are red because only red light can get through thicker atmosphere



[Not to scale!]

The Solar Constant

The **solar constant** is the amount of energy received per unit time on a surface of unit area at right angles to the sun's rays in the absence of the Earth's atmosphere. It is given by:

$$1370 \text{ J m}^{-2} \text{ s}^{-1}$$

Example

What area of solar panels would be needed in space to produce the same amount of power as Torness power station (1200MW) if the panels are 5% efficient.

Energy falling on panels = Area $\times$ solar constant

Energy produced = efficiency $\times$ energy falling on panels = efficiency $\times$ area $\times$ solar constant

$\Rightarrow$ Area = Energy produced / (efficiency $\times$ solar constant)

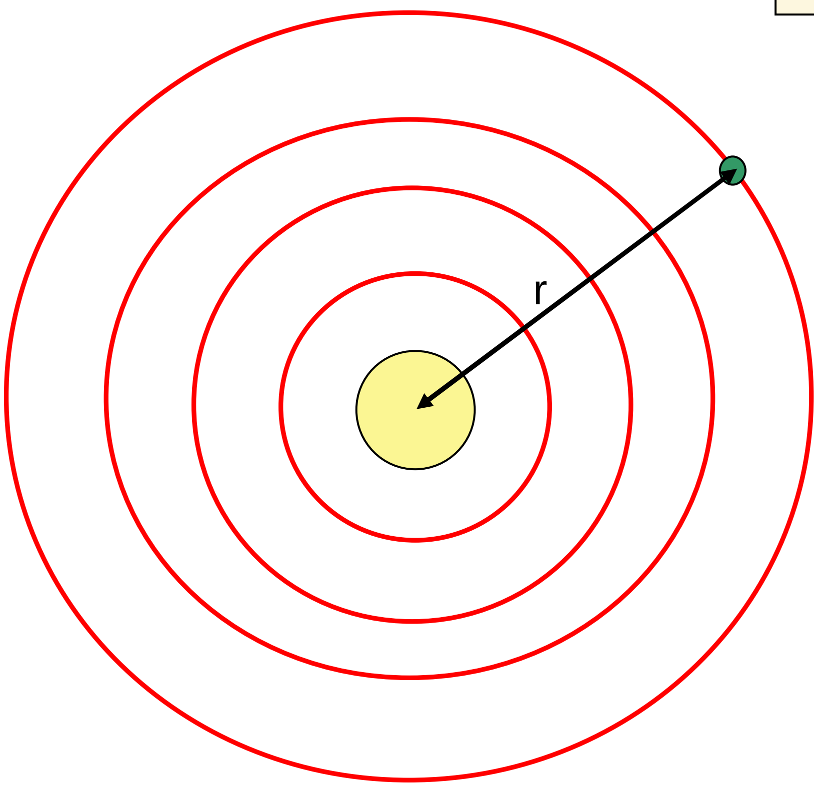
$= 1200 \times 10^6 \text{ J s}^{-1} / (0.05 \times 1370 \text{ J m}^{-2} \text{ s}^{-1})$ [$1\text{MW} = 10^6 \text{ W} = 10^6 \text{ J s}^{-1}$]

$= 17.5 \times 10^6 \text{ m}^2$ $= 4.2 \text{ km} \times 4.2 \text{ km}$

The solar constant will be different for other planets

If a source emits P Joules per second of light, then P Js^{-1} will pass through every spherical surface in one second. Since the size of these spheres gets bigger as we move away from the source (according the Area = $4 \pi r^2$) then the energy per second per square metre must decrease according to

$$I = P / 4 \pi r^2$$

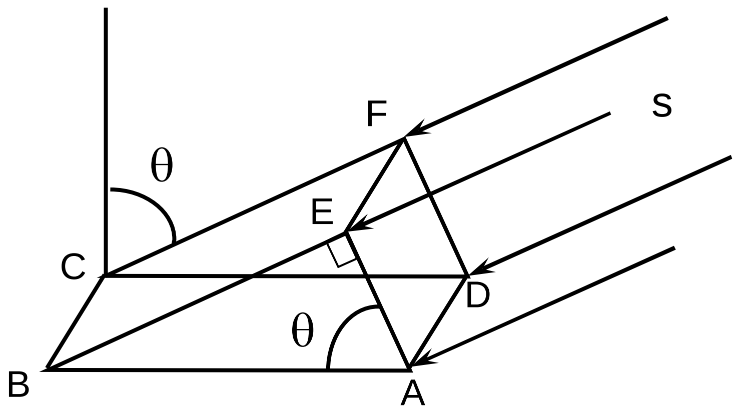


So if Saturn is 1.35 billion km from the Sun, what is its solar constant?

Remember, you can calculate the Sun's total energy output using the Stefan-Boltzmann Law

The Sun's energy on the Earth

The amount of energy per square metre hitting the Earth depends on latitude because of the angle of the sun's rays to the surface.



Consider a latitude where sunlight hits the ground at an angle θ as shown.

Let the solar constant be s, and distances AE=EF=1m. Since AEFD is a square of unit area, it will receive energy s per second, and it is this energy which will spread itself over ABCD on the ground.

Distance AB = AE/cosθ, so area ABCD = AEFD/cosθ = 1/cosθ

Energy hitting ABCD per second = s/ABCD = s cosθ

We have a lot of sunlight per square metre of ground at the equator and less towards the poles.

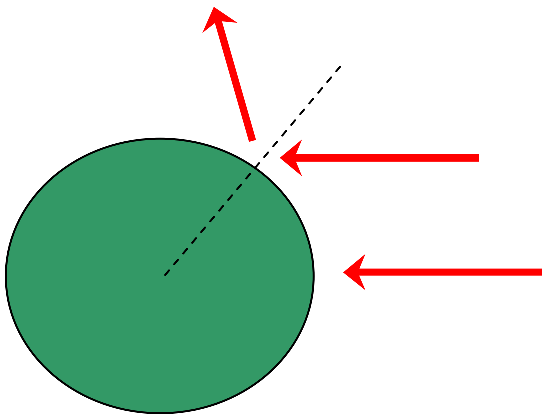
Glasgow is latitude 56^{o} North so it could at most get cos56^{o} = 0.56 of the sun the equator gets.

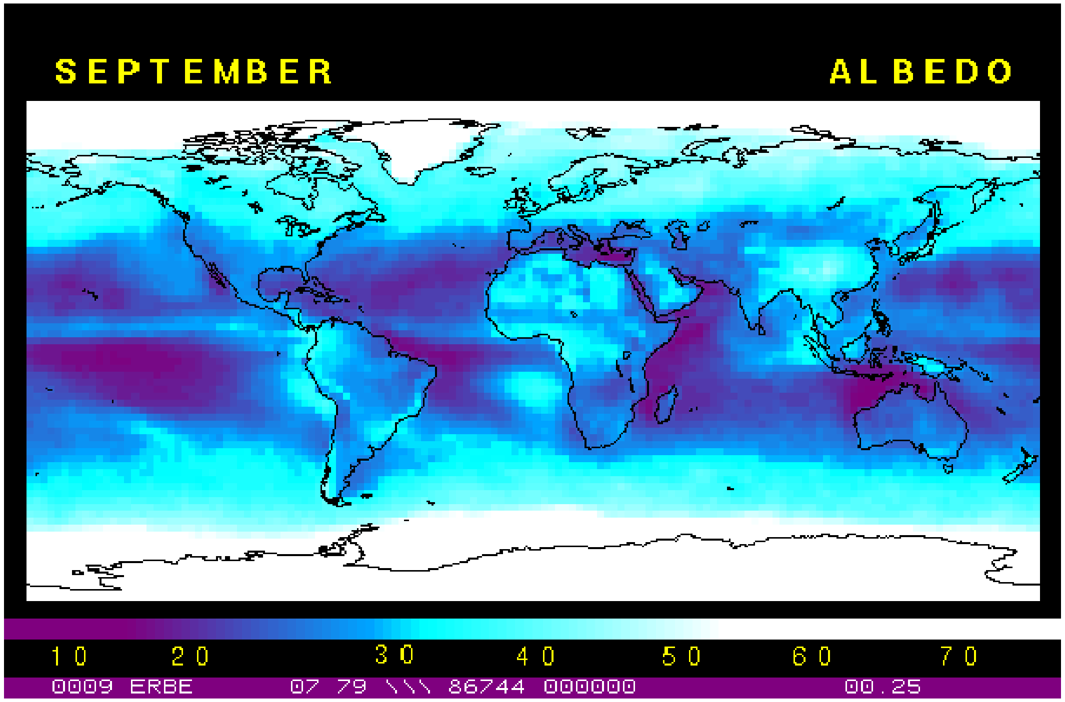
Albedo

Albedo is the proportion of incident energy which a planet reflects.

For the Earth, albedo varies with latitude. Light which hits at a glancing angle is more likely to be reflected.

Glasgow gets even less sun than we thought!





NASA Earth Radiation Budget Experiment

Albedo is also very dependent on cloud cover

The Earth's average albedo is about 0.3

The effective temperature of Earth as seen from space

Average temperature of Earth is constant

$$\Rightarrow \text{Energy input} = \text{Energy output}$$

Let the solar constant be s (1.37 kWm^{-2}), the albedo be a (0.3) and the radius of the Earth R

$$\text{Energy input per second} = \pi R^2 (1-a) s$$

$$\text{Energy output per second} = 4 \pi R^2 \sigma T^4$$

So,

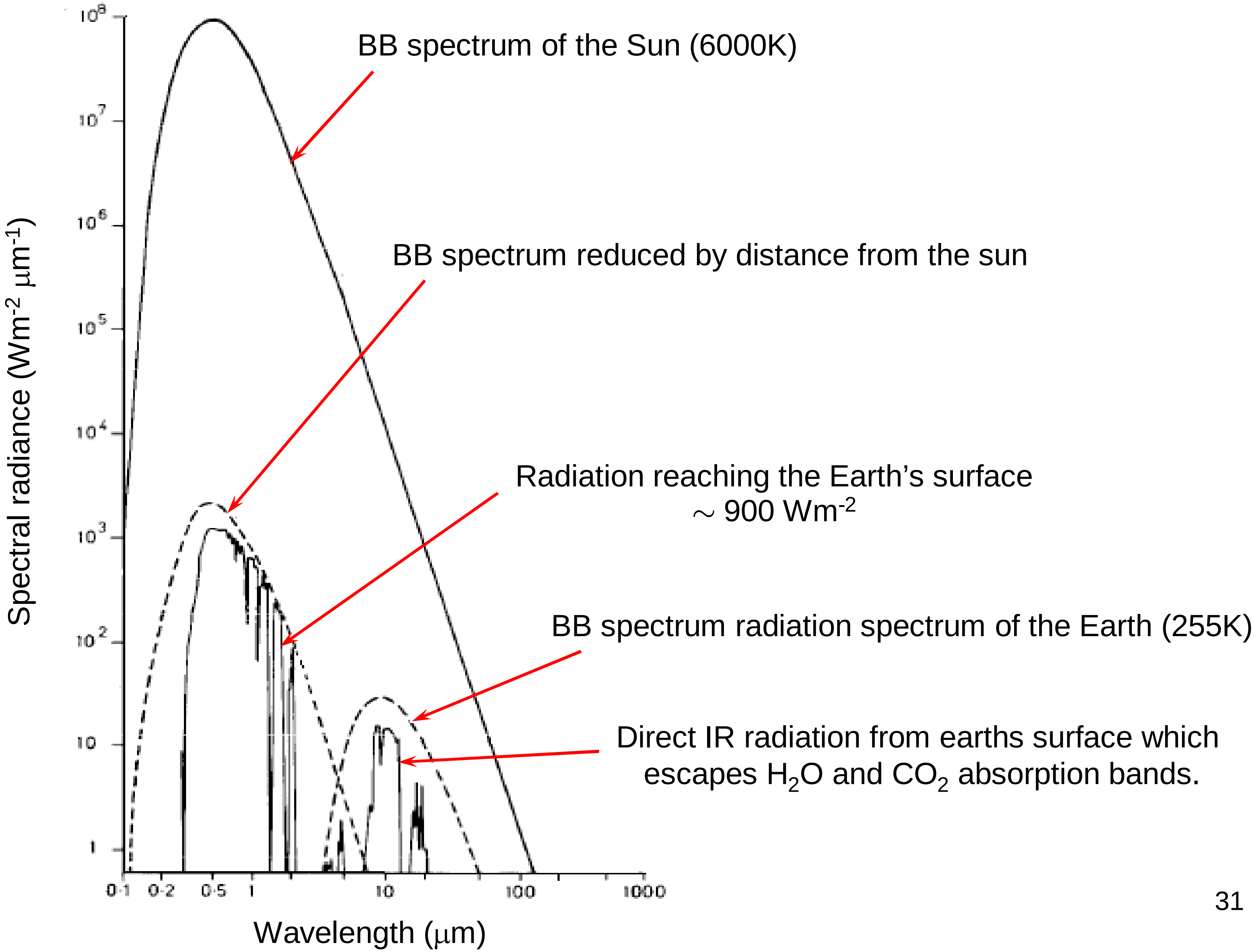
$$\pi R^2 (1-a) s = 4 \pi R^2 \sigma T^4$$

$$T^4 = s (1-a)/(4\sigma)$$

$$\boxed{T = 255K}$$

We can use the same method to estimate the temperature of planets.

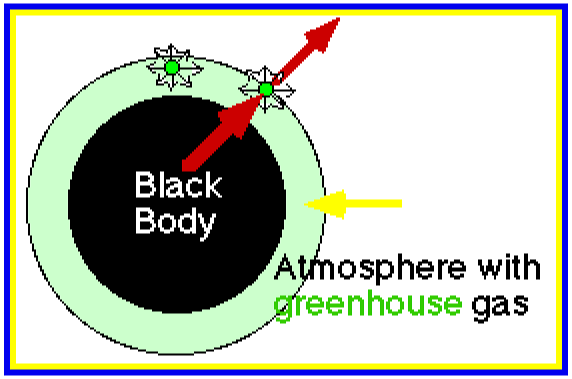
Note that this is not the same at the temperature on the surface of the Earth due to the Greenhouse effect…


BB spectrum of the Sun (6000K)
BB spectrum reduced by distance from the sun
Radiation reaching the Earth's surface
$\sim$ 900 Wm^{-2}
BB spectrum radiation spectrum of the Earth (255K)
Direct IR radiation from earths surface which escapes H_2O and CO_2 absorption bands.
Spectral radiance (Wm^{-2} µm^{-1})
Wavelength (µm)
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10
1
0·1
0·2
0·5
1
10
100
1000

The Greenhouse Effect

The average temperature on the Earth's surface is about 288K (15^{o}C) which is 33K warmer than the temperature seen from space. This difference is caused by the Greenhouse effect.

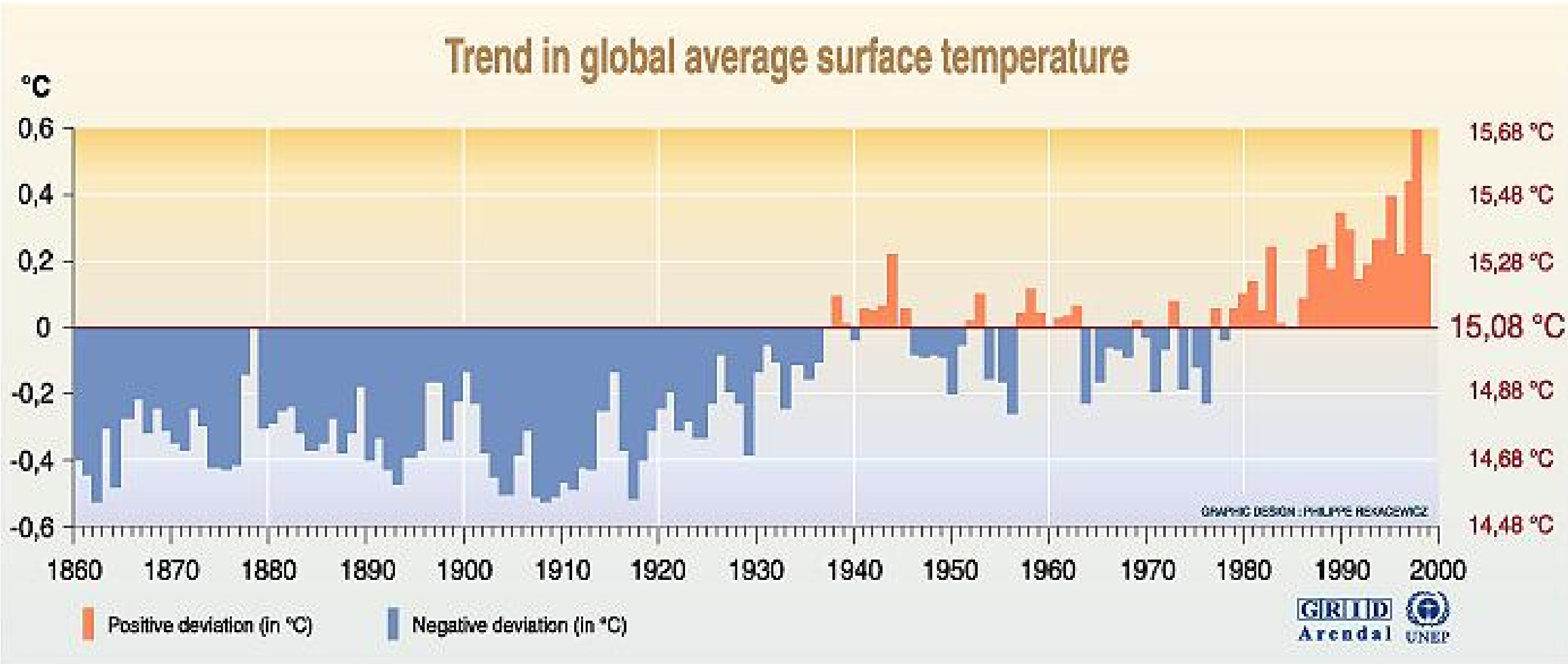
The Earth's (black-body) emission spectrum peaks in the Infra-Red at about 10μm (remember Wein's law $\lambda_{max} = a/T$), but except for a band around 8-12μm, much of this IR energy is absorbed by gases in the atmosphere. The absorbed IR is then re-emitted in all directions, half of which is redirected back to the Earth.



The Earth is warmer at the surface than we would see from space

Greenhouse gases which contribute most to the global temperature (in order of importance):

H_2O, CO_2, O_3, CH_4, CFCs.

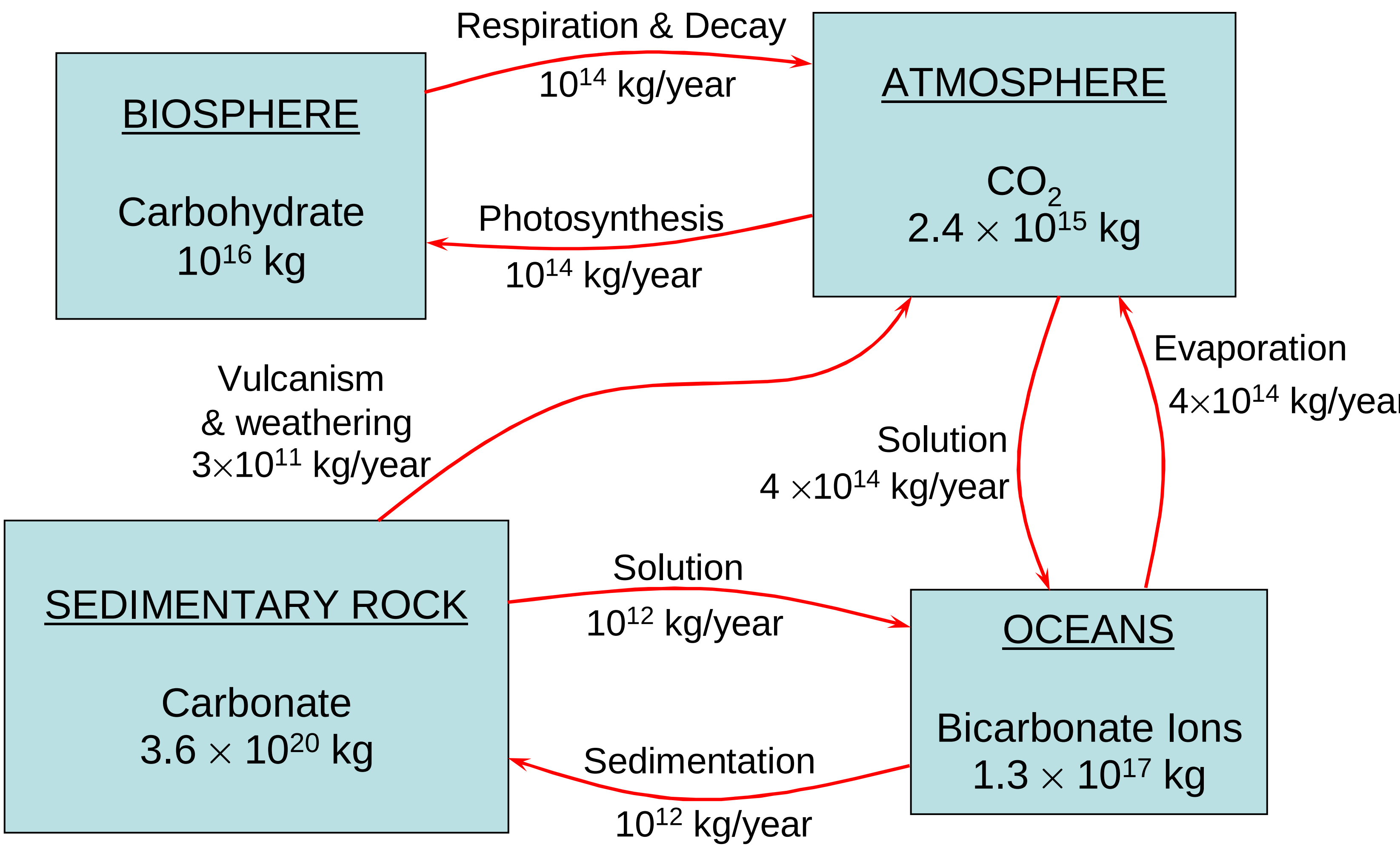


Source: School of environmental sciences, climatic research unit, university of East Anglia, Norwich, United Kingdom, 1999.

Man emits extra CO_2 into the atmosphere by burning fossil fuels (coal, oil & gas).

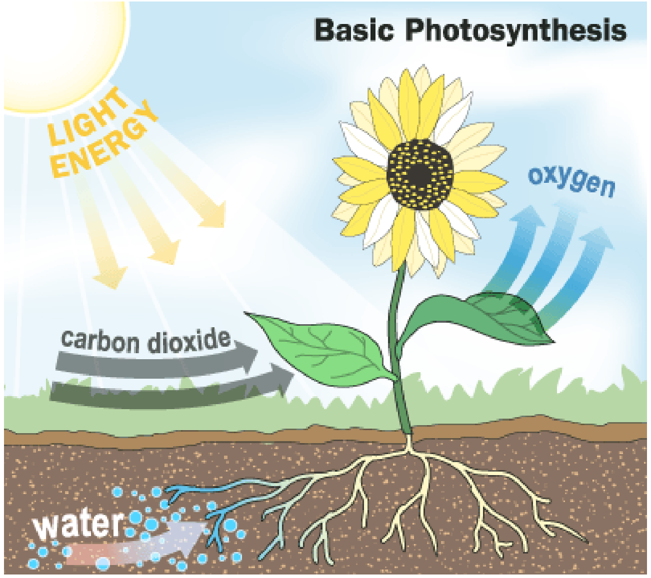
It is estimated that this has increased the CO_2 in the atmosphere by 20% in the last 140 years, which corresponds to a rise in temperature of 1K.

The Carbon Cycle



Burning fossil fuels adds 5.4×10^{12} kg / year

Photosynthesis



About half of the sun's energy hitting the Earth's surface is used in photosynthesis.

Light energy is trapped by chlorophyll in plants and used to convert CO_2 and water into glucose and oxygen.

$$6\ H_2O + 6\ CO_2 + \text{light energy} \longrightarrow C_6H_{12}O_6 + 6\ O_2$$

All oxygen in the atmosphere comes from this process.

The decay of organic matter and respiration remove O_2 form the atmosphere and replace it with CO_2

Absorption of CO_2 by the Ocean

The oceans form an important part of the Carbon Cycle. CO_2 is absorbed by the oceans and stored as bicarbonate ions. Cold water traps CO_2 more easily, leading to a feedback mechanism

increased CO_2 → larger greenhouse effect → warmer ocean

There are many other competing mechanisms:

more H_2O in atmosphere

more clouds ← higher albedo

less input energy

lower temperatures

icecap melts

lower albedo → higher temperatures

The effects of the oceans on global climate change is a very complex problem

3. Physics of The Atmosphere

The Composition of the Atmosphere

The Earth's atmosphere is about **200km** thick. Its approximate composition is:

N_2 (Nitrogen)	O_2 (Oxygen)	Ar (Argon)	H_2O (water vapour)	CO_2 (Carbon Dioxide)	Ne (Neon)	He (Helium)	O_3 (Ozone)
78.08%	20.98%	0.93%	0.001 – 1%	2.6×10^{-2}%	1.8×10^{-3}%	5.2×10^{-4}%	10^{-6}%

Over most of the atmosphere, these concentrations are constant, but near the very top of the atmosphere, radiation splits up molecules into atomic O, N, H and He.

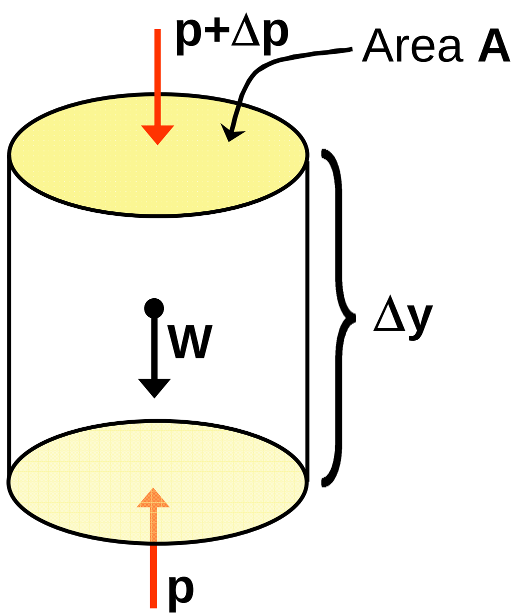
The atmosphere so contains **Aerosols**. These are very fine particles or large molecules ($\sim 10^{-8}$m) which are suspended in the atmosphere. They can be sea salt, silicates, organic matter, smoke particles, particles from volcanic eruptions etc. They act as centres of nucleation for rain drops and increase the albedo.

The variation of pressure with height

The **pressure** in the atmosphere changes depending on **how high** we are, and with the **temperature**.

- If we assume temperature is constant, how will pressure change with **height**?

Consider a **cylinder of air** of volume A Δy as shown:



The force on the cylinder due to gravity is the weight **W**. This is balanced by a difference in the pressure on the ends of the cylinder.

Let ρ be the density and **g** the acceleration due to gravity.

$$\left.\begin{array}{ll} & W = pA - (p+\Delta p)A = -\Delta p\, A \\ \text{But,} & W = mg = (\rho\, A\, \Delta y)\, g \end{array}\right\} \Delta p\, \cancel{A} = -(\rho\, \cancel{A}\, \Delta y)\, g$$

For a small **increase** in height the pressure **increases** by $\mathbf{\Delta p = -\rho\, g\, \Delta y}$

But the density changes with height too, so we are not quite finished.

For one mole of an ideal gas **pV = RT** $\Rightarrow$ **V = RT/p**

If the weight per mole is M, the density is **ρ = M/V = Mp/RT**

Using our previous result, the rate of **increase** of pressure with height is

$$\frac{\Delta p}{\Delta y} = -\rho g = -\left(\frac{Mp}{RT}\right) g = -\mu p$$

$$\mu = \frac{Mg}{RT}$$

$$\boxed{\frac{dp}{dy} = -\mu p} \quad \Longrightarrow \quad \boxed{p = p_o e^{-\mu y}}$$

height

Pressure at surface

Neglecting temperature change, pressure decreases with height **exponentially**.

Air pressure is usually measured in **millibars** (mb), where

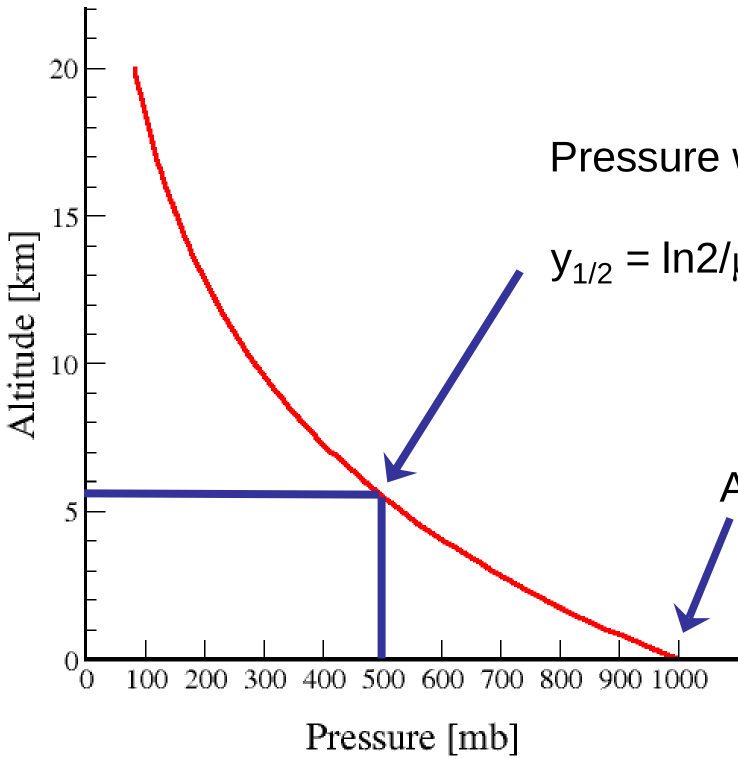
1000mb = 1 bar = 10^5 N/m^2

At ground level, the air pressure is usually around 1000mb.

- Let's estimate the value of μ for air

  Molecular weight M ≈ 29g mol^{-1}= 0.029kg mol^{-1} (O_2 = 32g mol^{-1}, N_2 = 28g mol^{-1})
  g = 9.81 ms^{-2}
  R = 8.31 J mol^{-1} K^{-1}
  T ≈ 273K

$$\text{So } \mu = \frac{Mg}{RT} = \frac{0.029\text{kg mol}^{-1} \times 9.81\text{ms}^{-2}}{8.31\text{Jmol}^{-1}\text{K}^{-1} \times 273\text{K}} = \underline{1.254 \times 10^{-4}\text{m}^{-1}}$$



Pressure with be half the sea-level pressure at a height

$y_{1/2}$ = ln2/μ = 0.693/1.254×10^{-4}m^{-1} = 5.5×10^{3}m = 5.5km

At sea-level (y=0) pressure is 1 bar = 1000 mb.

The variation of temperature with height

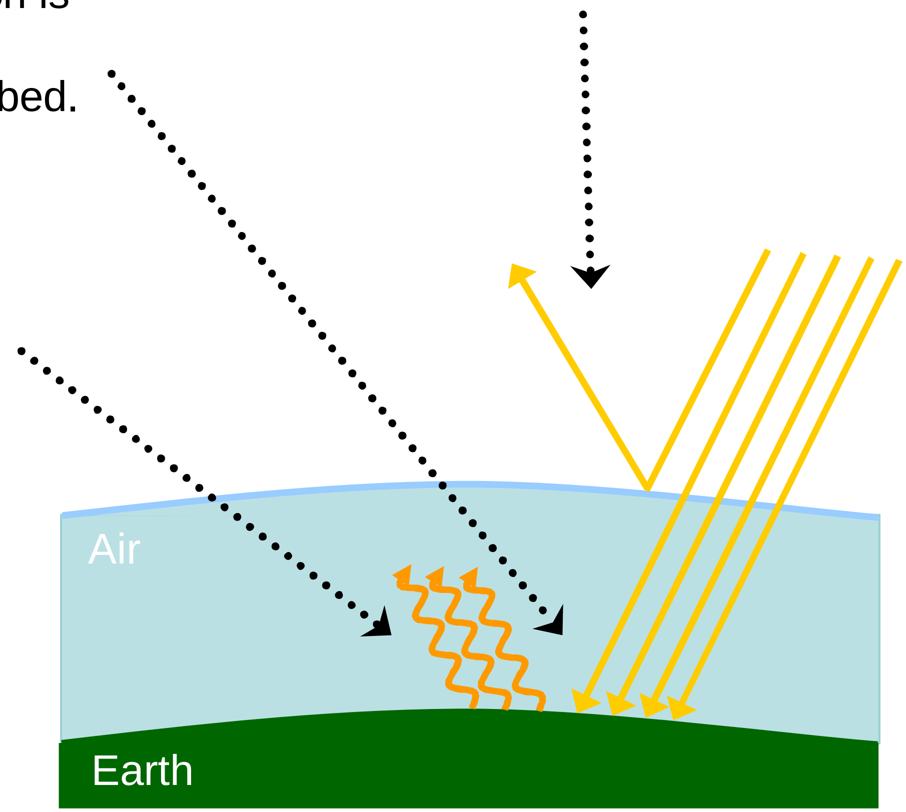
The variation of temperature with height depends on whether the air is **dry** or **moist**.

Dry Air

Some radiation from the sun is reflected by the atmosphere.

Very little of the remaining radiation is absorbed by the atmosphere
It reaches the ground and is absorbed.

This energy is released as heat, heating the atmosphere from below. The atmosphere is hottest close to the Earth and the temperature **decreases** with height.



The rate of decrease is known as the **lapse rate**.

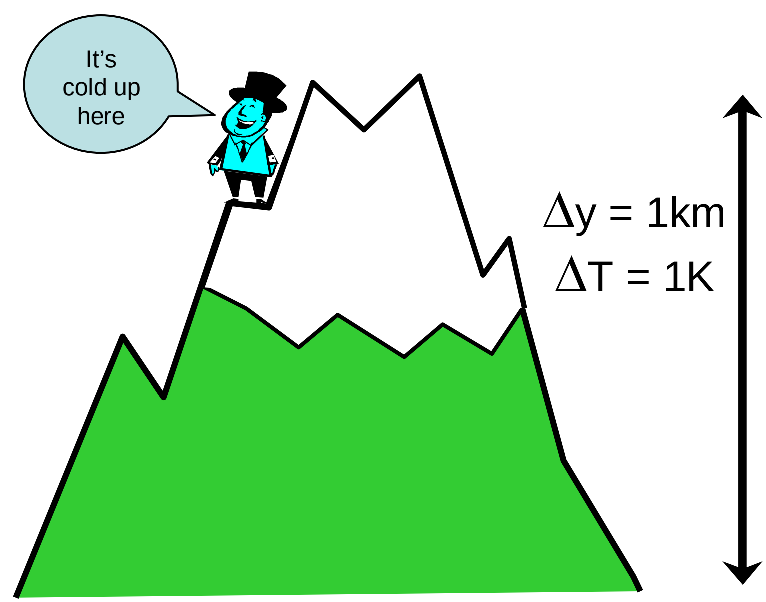
When air rises in the atmosphere, it gains potential energy and loses kinetic energy.

The temperature of a gas is a measure of the kinetic energy of the molecules in the gas
$\Rightarrow$ the temperature decreases.

This is assuming that no heat escapes the volume of gas, so it is an **abiabatic** process.

The laspe rate in this case is known as the **Dry Adiabatic Lapse Rate (DALR)**.

Its value is about **10K/km**, which means that the temperature will decrease by 10K if you climb a mountain which is 1000m high.

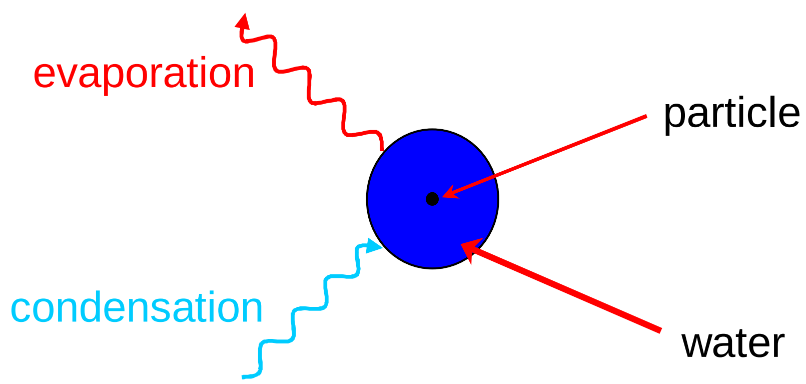


Moist Air

Air can contain water, both as vapour and as liquid drops. When it cannot hold any more water, it is said to be **saturated**.

The maximum amount of water depends strongly on temperature.

Liquid drops form around particles in the air: they can then get bigger by a process of **condensation**, or smaller by **evaporation**.

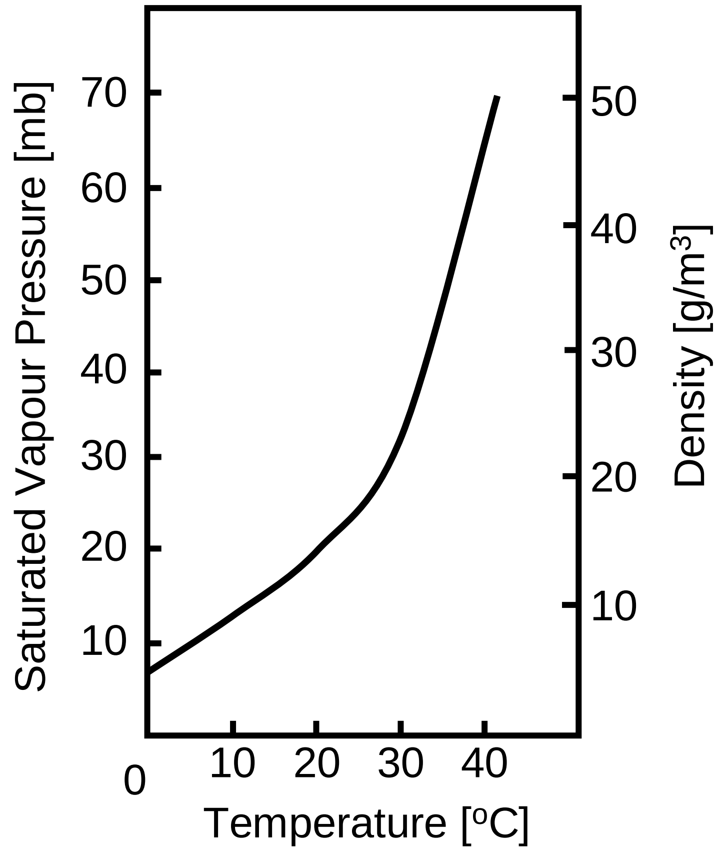


Condensation: how fast water molecules condense depends on water vapour pressure.

Evaporation: how fast water molecules leave the water drop depends on the temperature.

$\Rightarrow$ Clouds form when the temperature drops

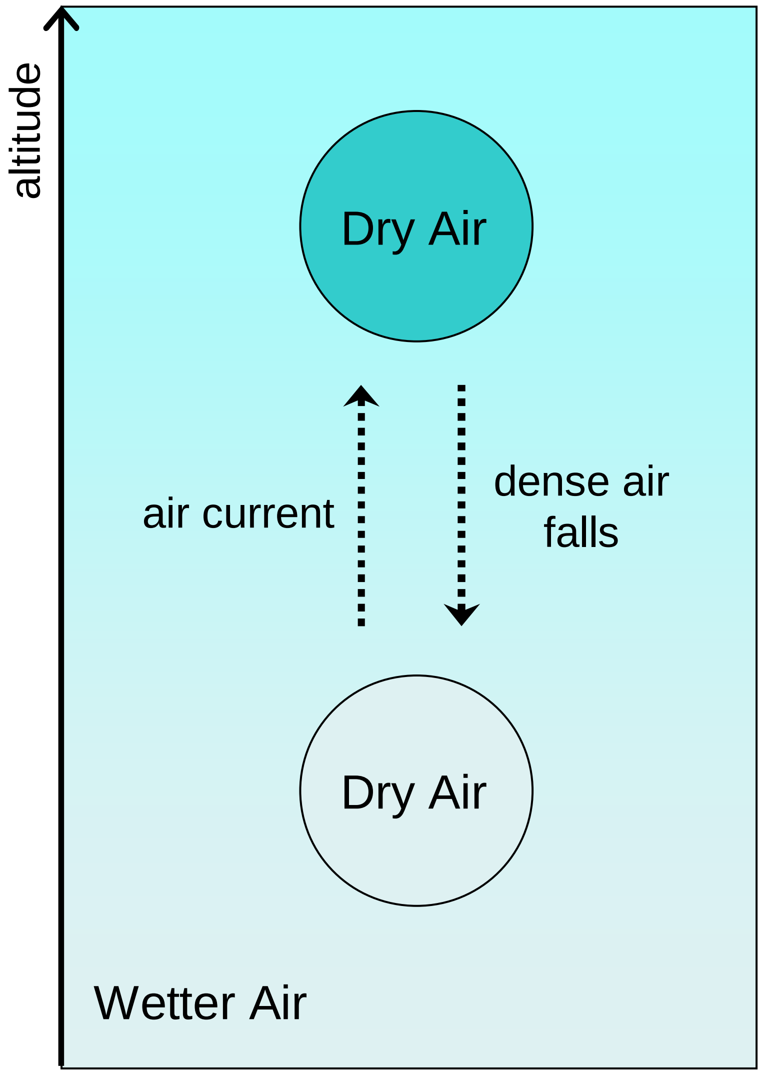
The **saturated vapour pressure** is the pressure of the vapour phase of a substance when it is in equilibrium with the liquid phase. The **relative humidity** is the **partial pressure of water vapour in the air** divided by the saturated vapour pressure (usually expressed as a percentage). When the relative humidity is 100%, the air is saturated.



As wet air rises, its temperature will lower and it will become saturated. Water droplets will start to form, releasing **latent heat** (heat released during the phase transition from water vapour to liquid). This will heat the air so that the overall lapse rate is lower than for dry air.

This is known as the **Saturated Adiabatic Lapse Rate (SALR)**. Since the saturated vapour pressure depends on temperaure, the SALR does too:

Temperature	15^{o}C	0^{o}C	-20^{o}C
SALR (K/km)	5	6.2	8.6

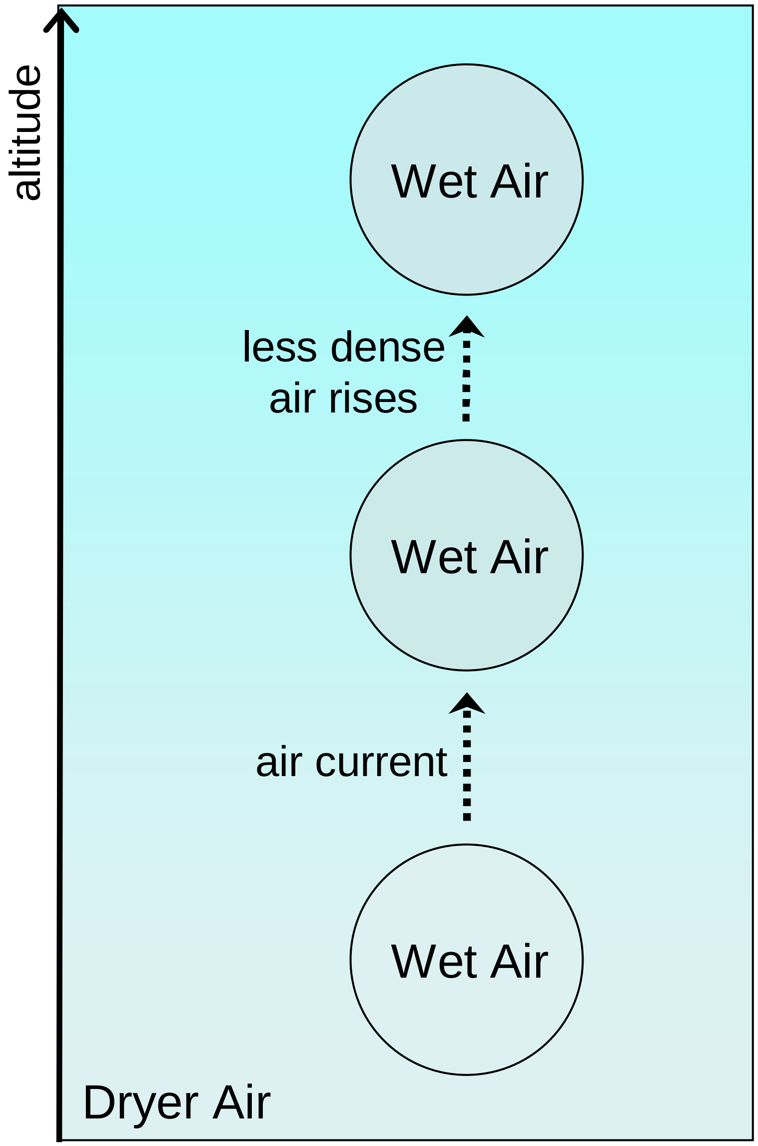


Imagine a volume of **dry air** surrounded by **wetter air**

If an air current pushes the dry air up, the dry air will become **colder** at a rate of 10K/km (DALR) while the wetter air will become **colder more slowly** (at the appropriate SALR).

The dry air becomes more **dense** than the surrounding air and **falls back down**.

Dry air is stable to weak air currents



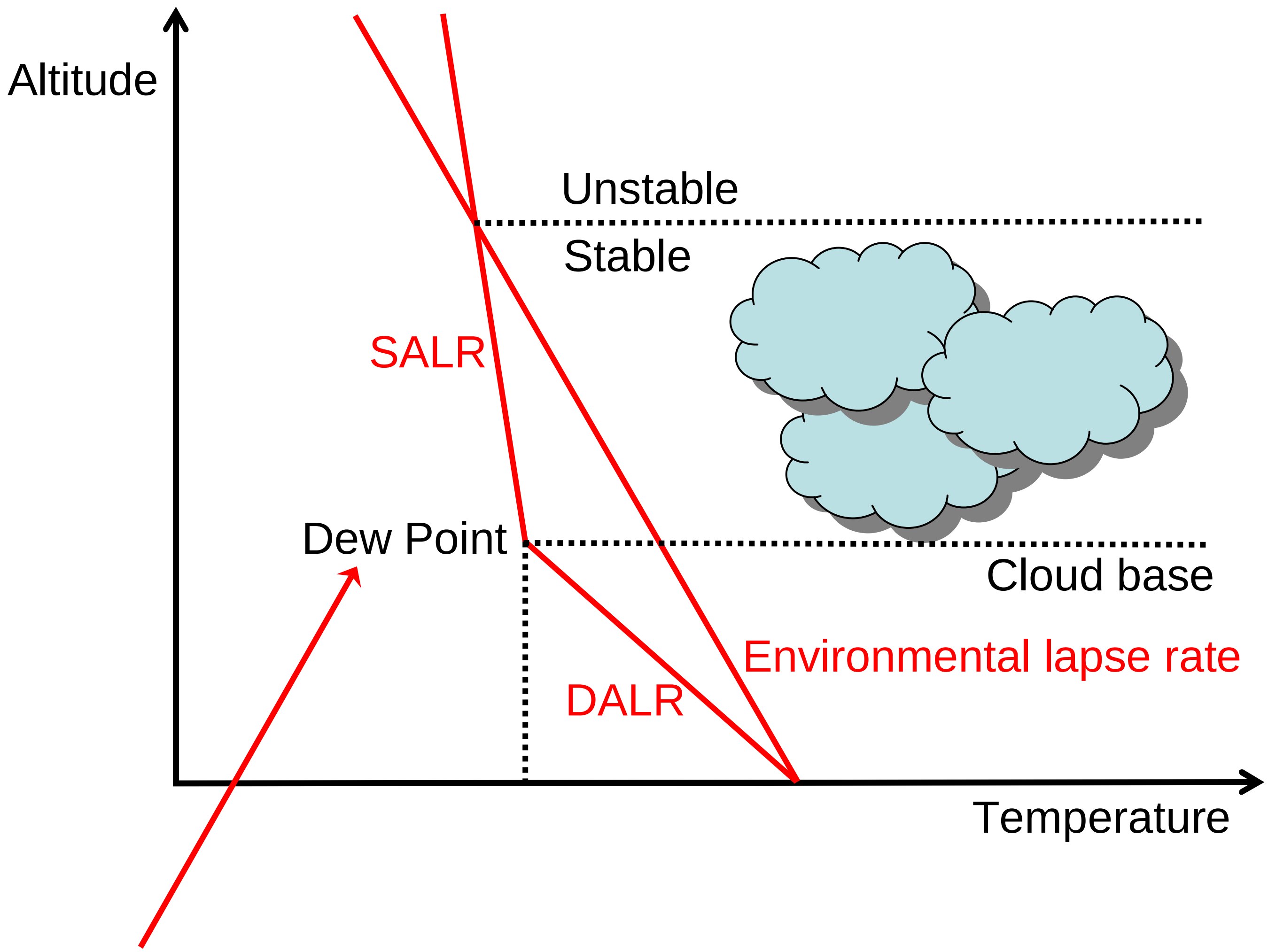
Now imagine **wet air** surrounded by **dryer air**.

If an air current pushes the wet air up, it will become **colder at a slower rate** (SALR) than the surrounding dryer air.

The warmer wet air is then **less dense and rises**. The temperature difference will become greater and it will continue to rise.

Wet air is unstable to air currents.

This results in dense clouds, heavy rain and thunderstorms.



At the **Dew point**, the partial pressure of water vapour in the air is equal to the saturated vapour pressure – above this, the water vapour will condense.

Layers of the Atmosphere

The five regions of the atmosphere are given names ending in **–sphere**, while the upper boundary of each region are given same suffix but with the ending **–pause**.

The main regions are (in order of ascending height):

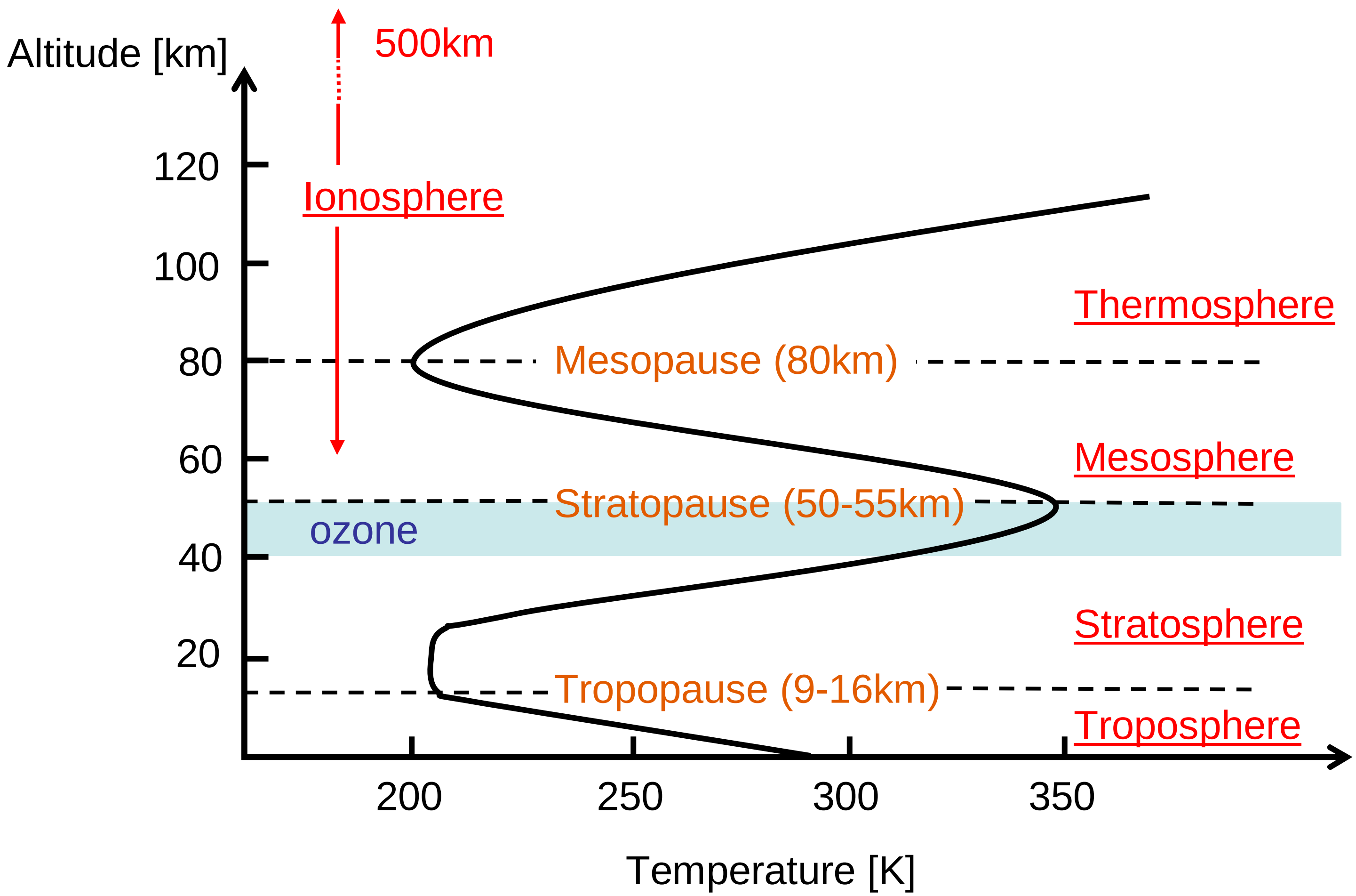
Troposphere: This is the layer in contact with the Earth. Life and weather are largely confined to this region. Our previous discussions have been concerned with this region.

Stratosphere: The stratosphere contains **ozone**, which absorbs radiation, causing a **positive temperature gradient** (i.e. it gets hotter with increasing altitude). This makes conditions very stable and confines "weather" (instabilities) to the troposphere.

Mesosphere: This region has very low pressure (1mb) making the density too low to absorb much solar energy. Therefore, temperature decreases with increasing altitude.

Thermosphere: Like the thermosphere, density is very low here. However, the density is so low that very little energy is required to increase temperature and temperature rises with altitude. UV radiation causes N, O, H and He to be present in atomic form, absorbing wavelengths of light < 200nm.

Ionosphere: Wavelengths < 120nm cause ionisation of atoms, forming positive ions and releasing free negative electrons. Low density makes recombination unlikely, so a permanent posulation of ions persists (hence the name iono-). Short wave radio signals can be reflected from the ionosphere. The ionosphere overlaps with the lower layers.



Altitude [km]
500km
120
Ionosphere
100
Thermosphere
80
Mesopause (80km)
60
Mesosphere
Stratopause (50-55km)
ozone
40
Stratosphere
20
Tropopause (9-16km)
Troposphere
200
250
300
350
Temperature [K]

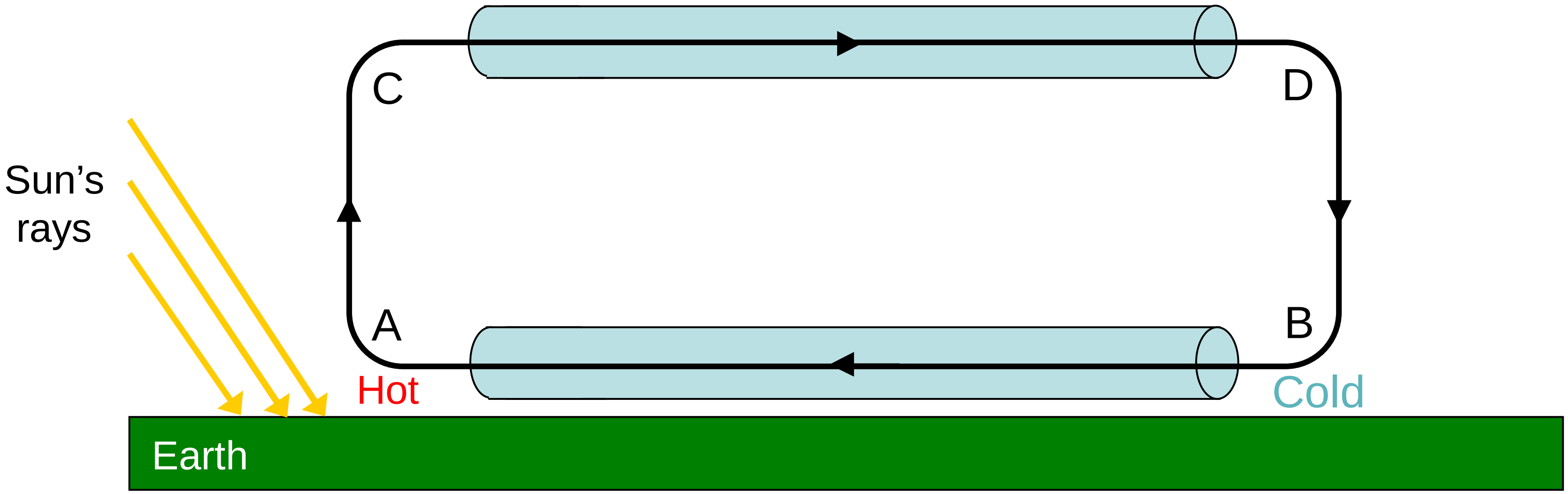
The General Circulation of the Atmosphere

The dependence of **albedo** on latitude and the **angle of the sun's rays to the ground** means that equatorial regions receive much more of the sun's energy than the poles.

(In fact, the maximum heating effect is at about 20^{o} north and south, since the sun's rays are nearly vertical overhead for three months rather than one at the equator.)

Pressure differences $\Rightarrow$ movement of heat from the equator to the poles

The Hadley Cell



Suppose the ground at a **point A is heated by the sun**, making the air above it warmer than that above **point B which is cold and dense**.

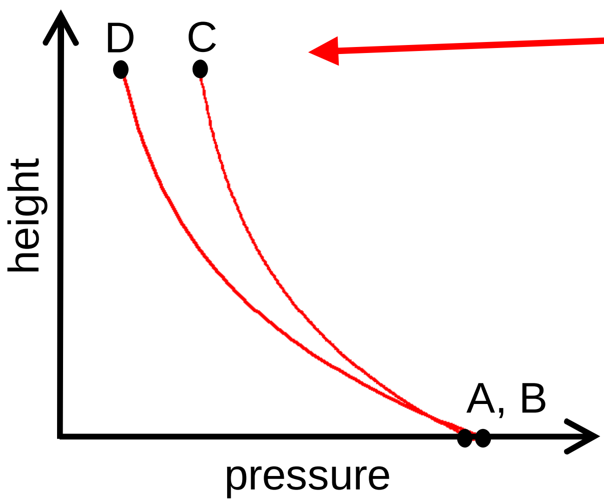
Pressure varies with height according to

$$p = p_o e^{-\mu y} \quad \text{with} \quad \mu = \frac{Mg}{RT}$$

Above A, T is high and μ is small $\Rightarrow$ pressure falls off slowly

Above B, T is low and μ is high $\Rightarrow$ pressure falls off quickly

At ground level, pressure is fairly constant, so pressures at A and B are nearly the same.



Pressure at C is higher than pressure at D

$\Rightarrow$ winds flow from C to D

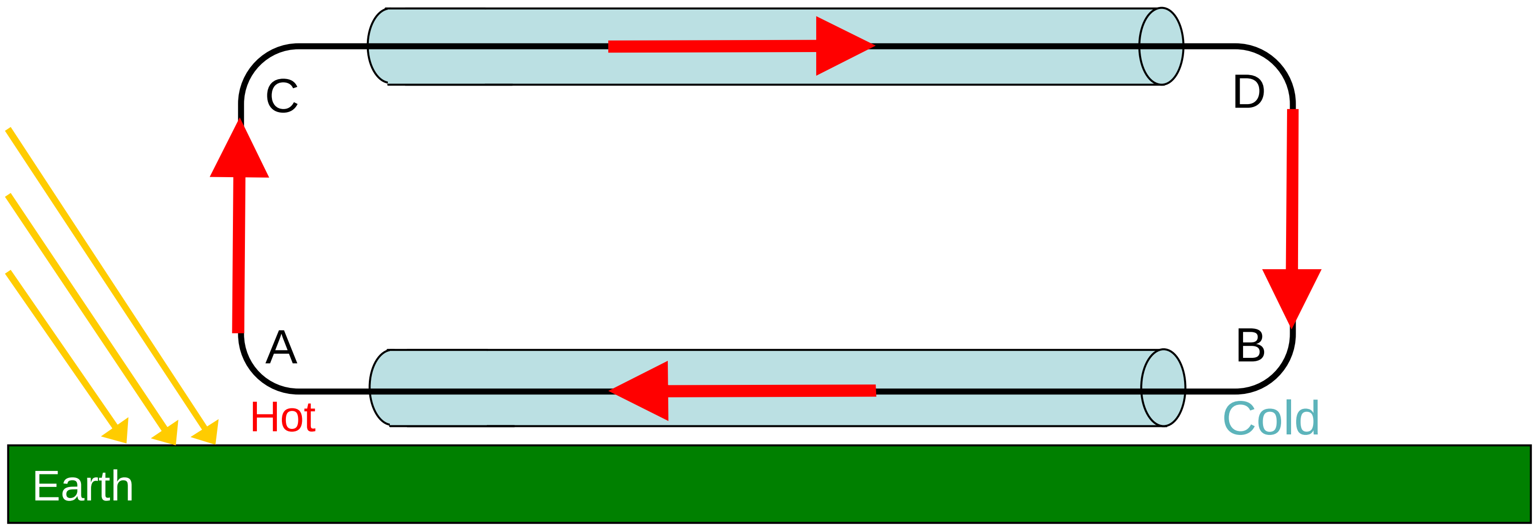
Sun’s rays heat Earth at A, which in turn heats the air above

Hot air at A rises to C

Pressure difference between C and D causes wind to flow from C to D

Air cools and falls to B

Conservation of mass forces wind to flow from B back to A



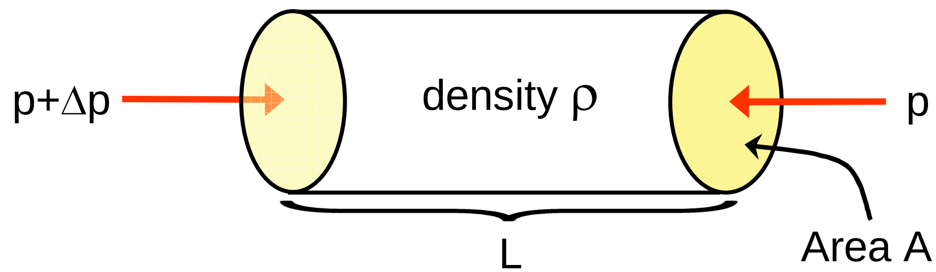
Low pressure $\longrightarrow$ low density

So the mass of air in the cylinder between C and D is less than that between A and B

Since the amount of air is conserved, the mass of air moving from C to D in unit time must be the same as that flowing from B to A $\Rightarrow$ Air must travel faster from C to D

$\Rightarrow$ Winds at **high altitude** are much **higher velocity** than at ground level

Consider a (short) cylinder of air, of mass m, density ρ and length L, with end area A



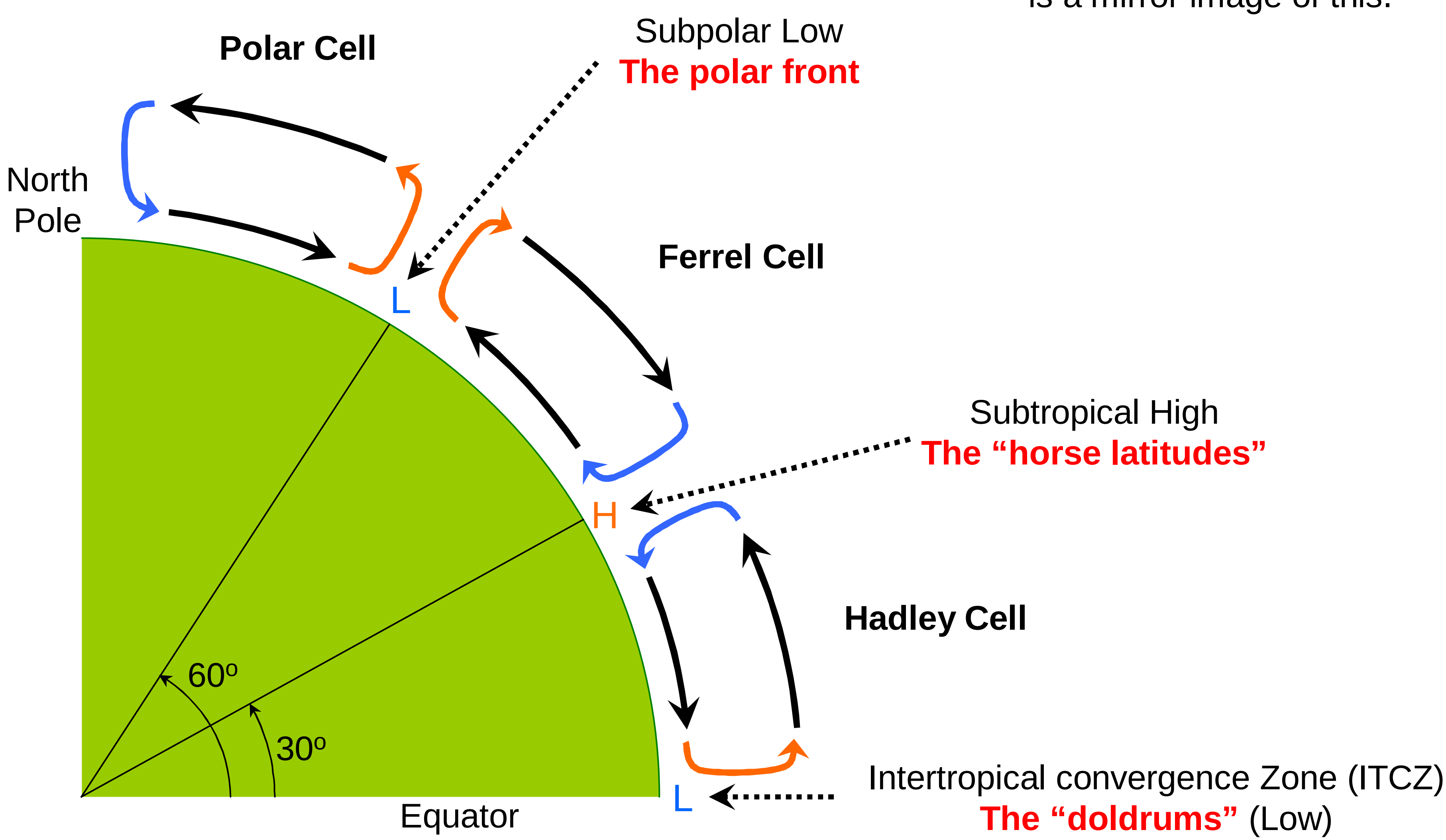
Newton's second law is **F=ma** (force = mass $\times$ acceleration)

$$\Rightarrow a = F/m = A \Delta p / (\rho L A) \propto \Delta p/p \qquad (\text{recall } \rho = Mp/RT)$$

Between A and B (ground level) $\Delta p/p \approx 10\text{mb}/1000\text{mb} = 1\%$,
Between C and D (top of the troposphere) $\Delta p/p \approx 4\%$

The three cell model

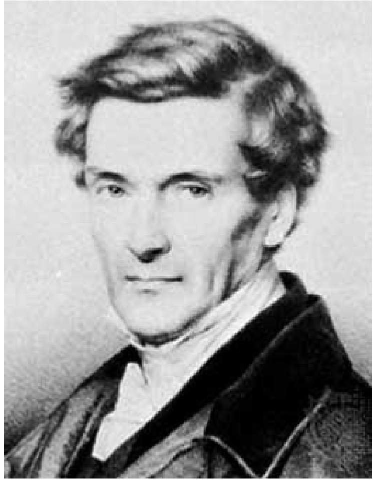
The southern hemisphere is a mirror image of this.



The Coriolis Force

Air movements will be modified by the effects of the Earth's rotation – the **Coriolis** force.

To an observer looking down on the Earth, all objects on to the Earth's surface are actually moving eastwards.



Gustave-Gaspard Coriolis 1792-1843

Radius of the Earth, $R = 6.4\times10^{6}$ m

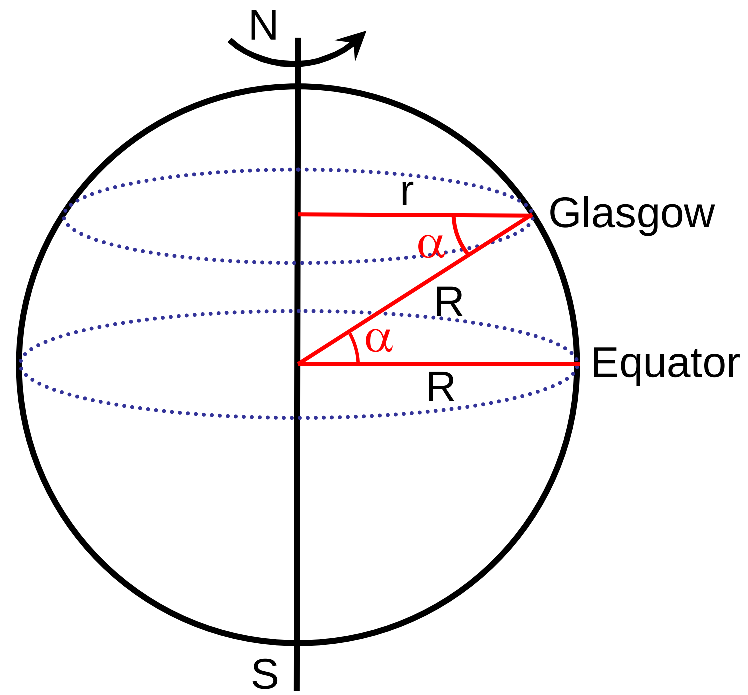
Earth's angular velocity, $\omega = 2\pi$ / 1 day = $2\pi / (24 \times 60 \times 60\ \mathrm{s}) = 7.27\times10^{-5}\ \mathrm{s}^{-1}$

So the velocity of a point on the equator is $v = \omega R = 6.4\times10^{6}\mathrm{m} \times 7.27\times10^{-5}\ \mathrm{s}^{-1}$ = **$465\ \mathrm{ms}^{-1}$**

[Alternatively, in 1 day the point must move a distance $2\pi R$, so must travel at a speed of $2\pi R$ / 1 day = $2\pi / (24 \times 60 \times 60\ \mathrm{s}) = 465\mathrm{ms}^{-1}$]

But this velocity depends on latitude – Glasgow need not move so far in one day

How fast a point moves east depends on its latitude.



Latitude of Glasgow, $\alpha \approx 56^o$

So its distance from the axis of the Earth is,

$r = R \cos 56^o \approx =3.56 \times 10^6$m

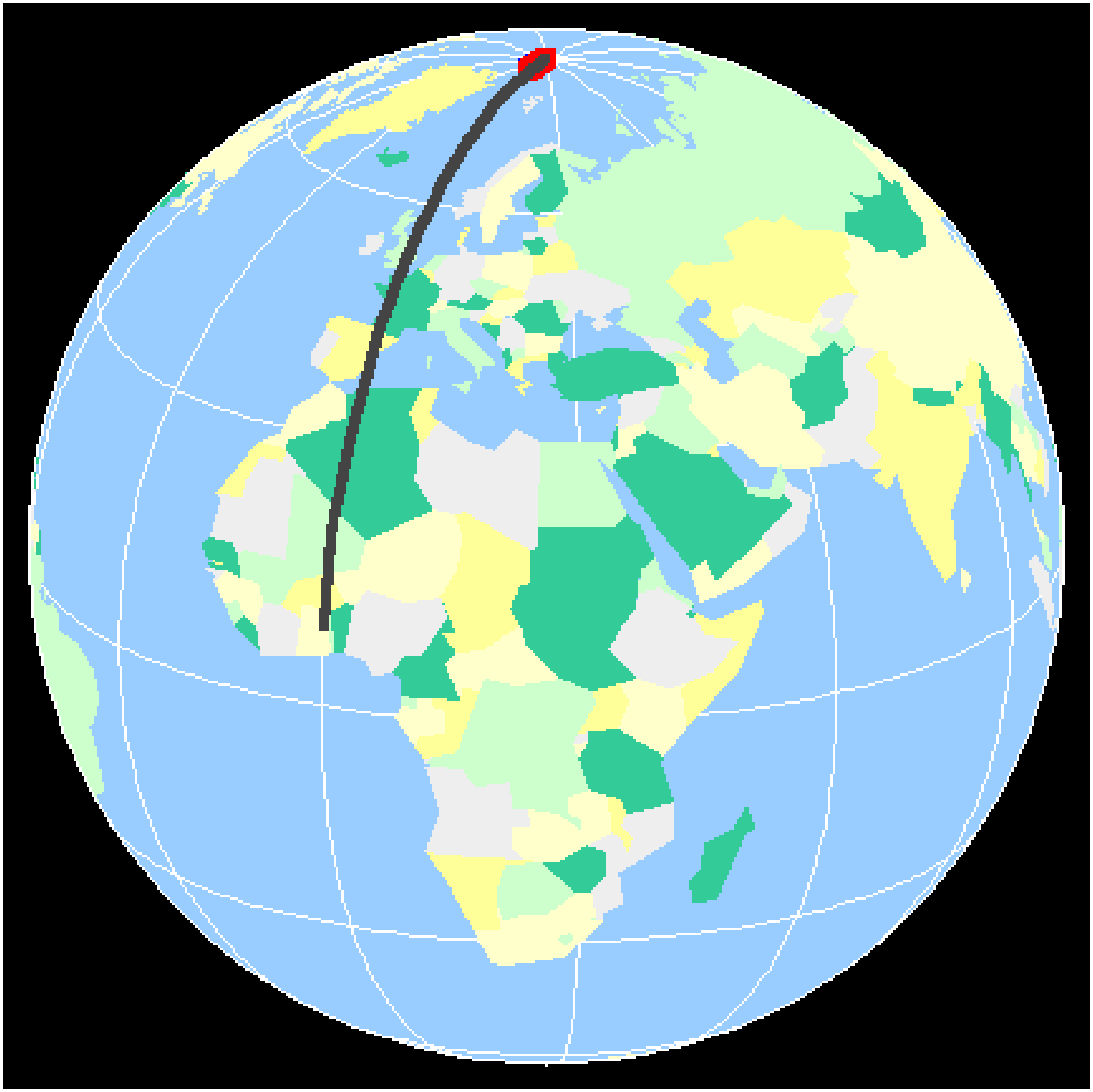
Glasgow need only travel $2\,\pi\, r$ in one day,
so moves much more slowly,

$v = 2\pi\, r\, /\, (24 \times 60 \times 60\ s) \approx$ **260 ms^{-1}**

Now imagine an object moving north from the equator. At the equator it is moving due north, so also has a component of velocity 465ms^{-1} to the east from the Earth's rotation. As it moves north it keeps this velocity component but by the time it reaches Glasgow this eastward velocity is more than that for a stationary object. Therefore the object will be moving eastwards with a velocity (465-260)ms^{-1} = 205ms^{-1}.

> Objects moving away from the equator will be "pushed" to the East.
> Objects moving toward the equator will be "pushed" to the West.

The is the **Coriolis Force**.



The coriolis force is a **force** because to an observer stationary on the Earth, the object is **accelerated**.

Newton's second Law: F = ma   Accelerations are caused by forces.

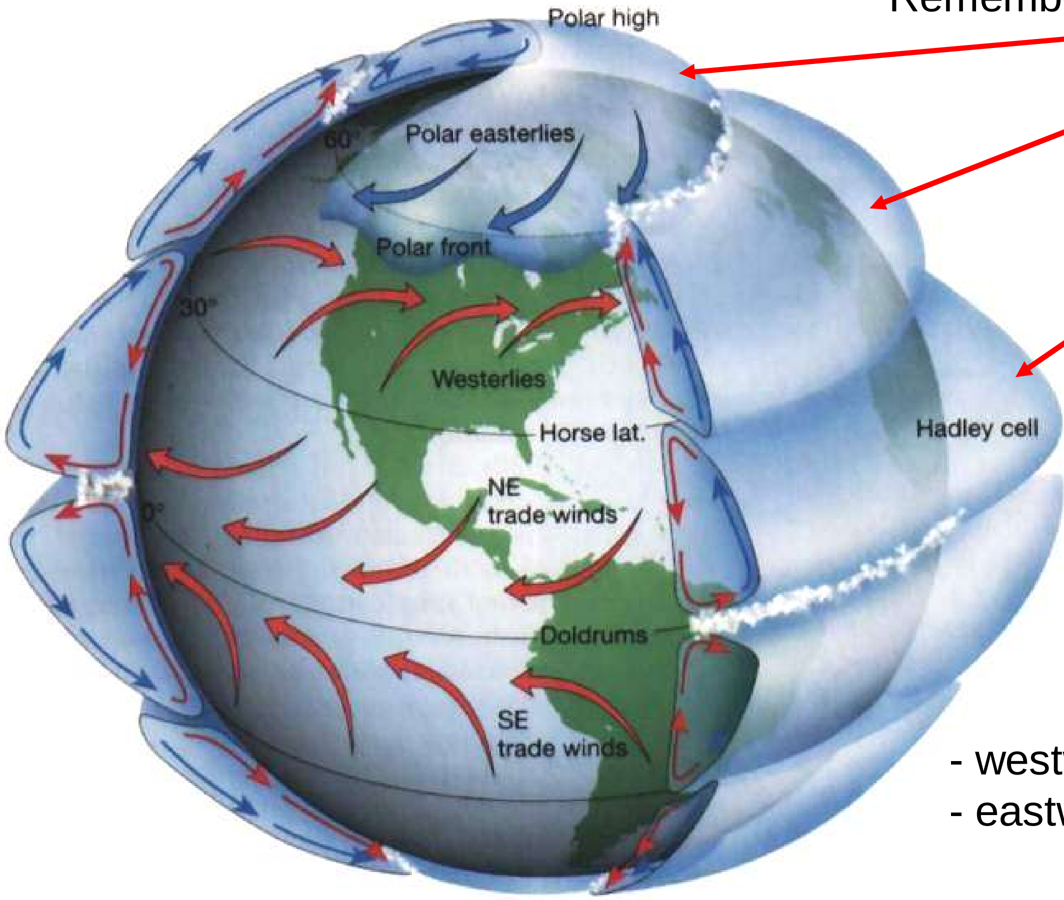
$$\boxed{F = 2m\vec{v} \times \vec{\omega} = 2mv\omega \sin\alpha}$$

However, it is not a fundamental force but is a **"fictitious"** force caused by the observer being in an accelerating frame of reference (a non-inertial frame).

Similar effects can be seen in other accelerating frames, eg a roundabout

This Coriolis force effects global winds:

Remember the Polar, Ferrel and Hadley cells.



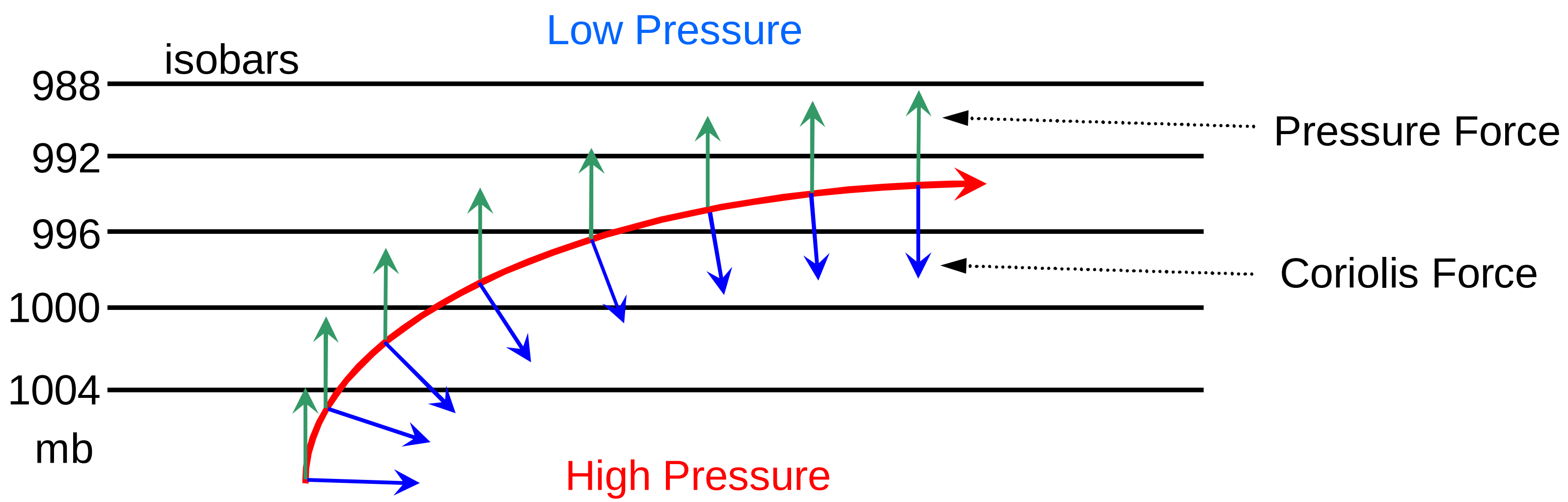
Air moving towards the lows at the **Doldrums** and **Polar Front** will be rotated by the Coriolis force

- westwards for winds towards the equator
- eastwards for winds away from the equator

Note: names of winds denote where they come from.

The Geostrophic Wind

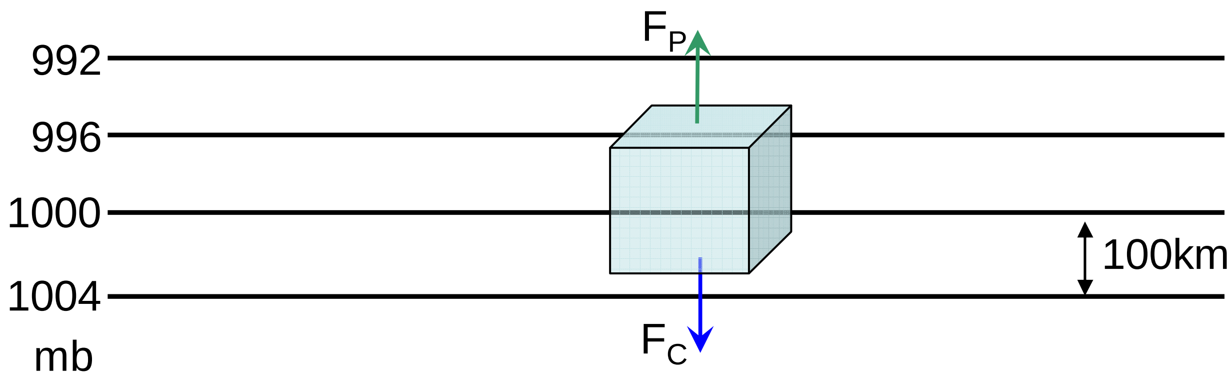
The Coriolis force can have other effects too. For example, it may balance the pressure gradient between high and low pressure area, causing air to flow along isobars. This is known as a **Geostrophic wind**.



The Coriolis force rotates the wind until the coriolis force and pressure forces are balanced.

definition of a geostrophic wind

Let’s estimate the speed of a geostrophic wind from a weather chart.



Imagine the weather over Glasgow showing isobars every 4mb with a distance of 100km between them.

The pressure difference across on a cube $(1\text{m})^3$ will be $4\text{mb}/100\text{km} \times 1\text{m} = 4\times10^3\ \text{Nm}^{-2}$
So the pressure **force** on the cube is

$$F_P = 4\times10^3\ \text{Nm}^{-2} \times \text{Area of } 1\text{m}^2 = 4\times10^{-3}\ \text{N}$$

This is balanced by the Coriolis force, $F_C = 2\ m\ v\ \omega\ \sin\alpha$, so that $F_P = F_C$

So the wind’s velocity is:

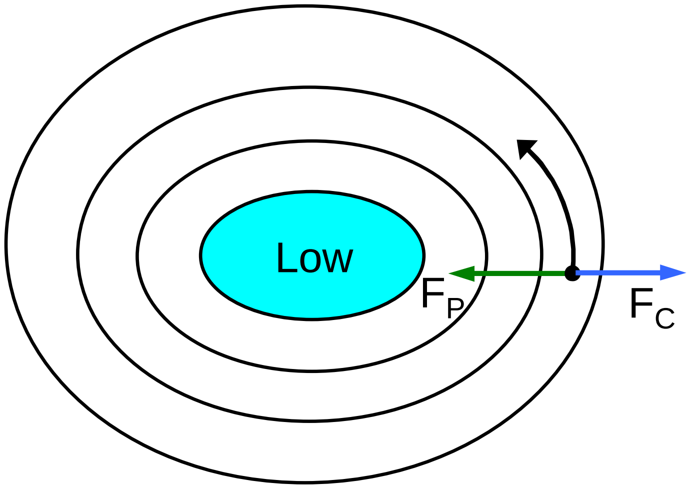
$$v = F_P / (2m\omega\sin\alpha) = 4 \times 10^{-3}\ \text{N} / (2 \times 1.3\text{kg} \times 7.27\times10^{-5}\text{s}^{-1}\ \sin 56^o) = 26\text{ms}^{-1}$$

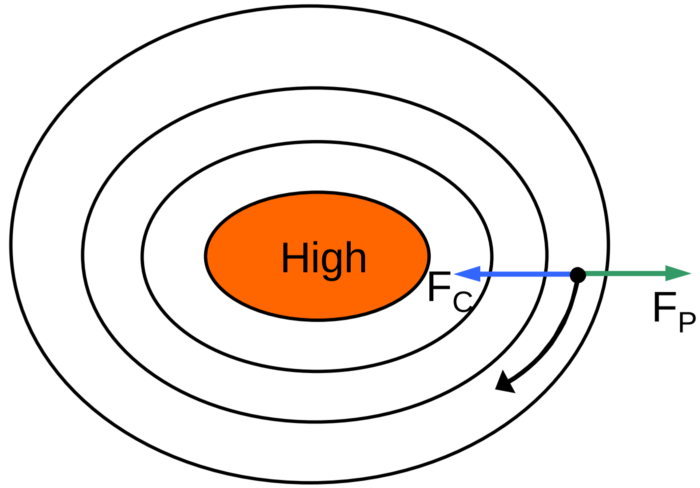
This is about 60mph.

mass of 1m^3 of air

Non-geostrophic winds

If the pressure force and the Coriolis force are not perfectly balanced the wind is said to be **non-geostrophic**. This often happens when we have localised low and high pressure areas.



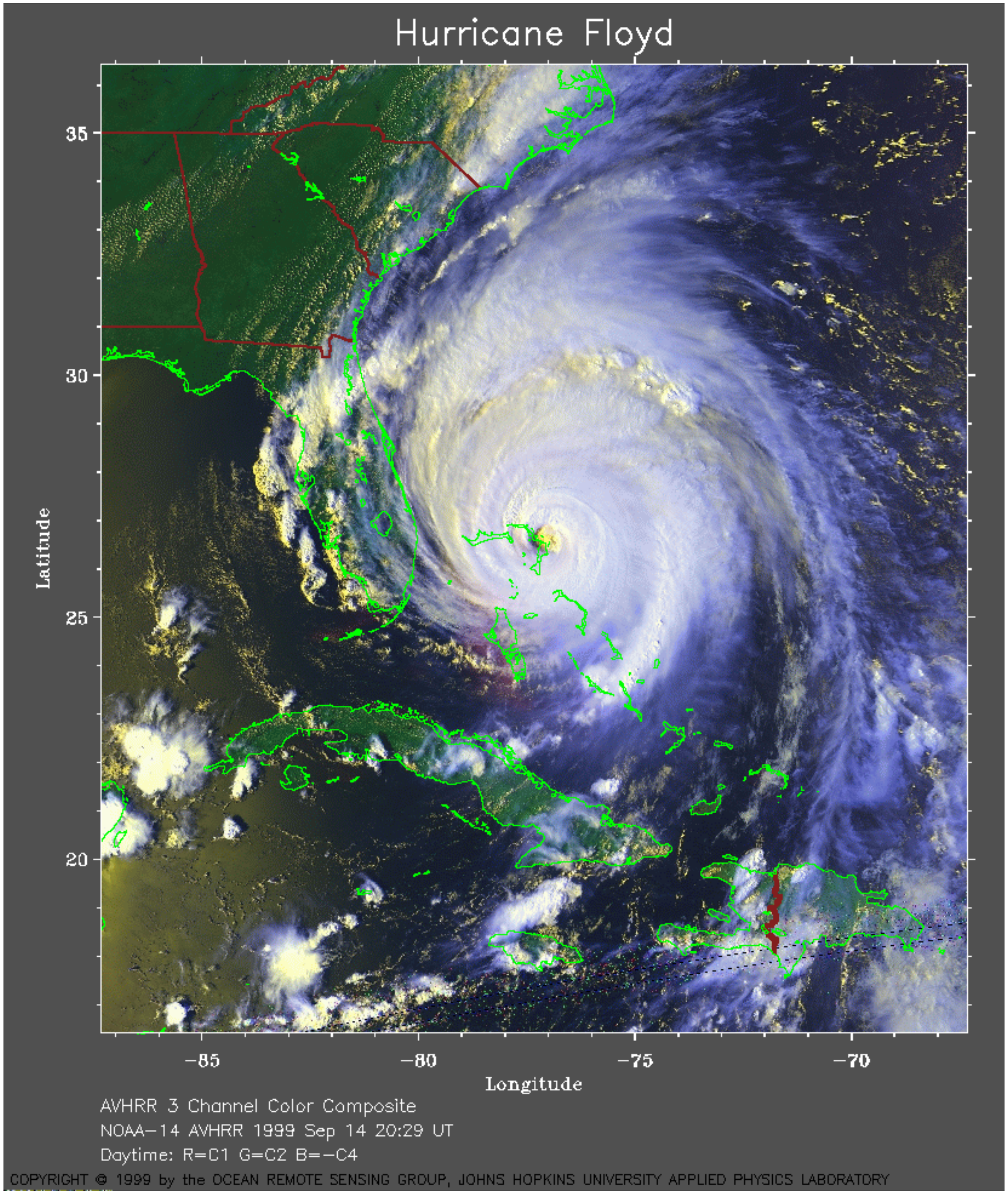


For **low pressure** region, the pressure force acts **inwards**. It may be partially balanced by the Coriolis force acting **outwards**, but if the **pressure force** is greater we will get circular motion.

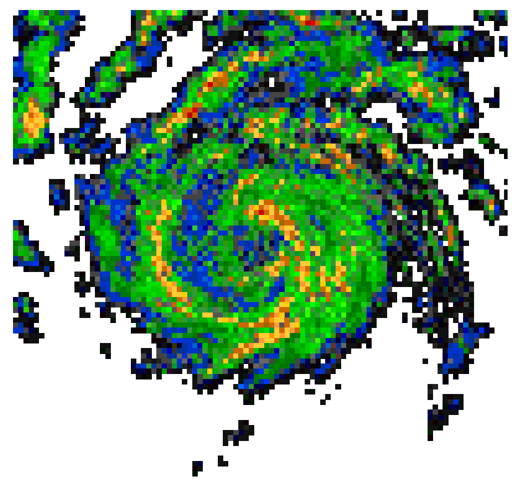
For a **high pressure** region, the pressure force acts **outwards**. It may be partially balanced by the Coriolis force acting **inwards**, and if the **Coriolis force** is greater we will get circular motion.

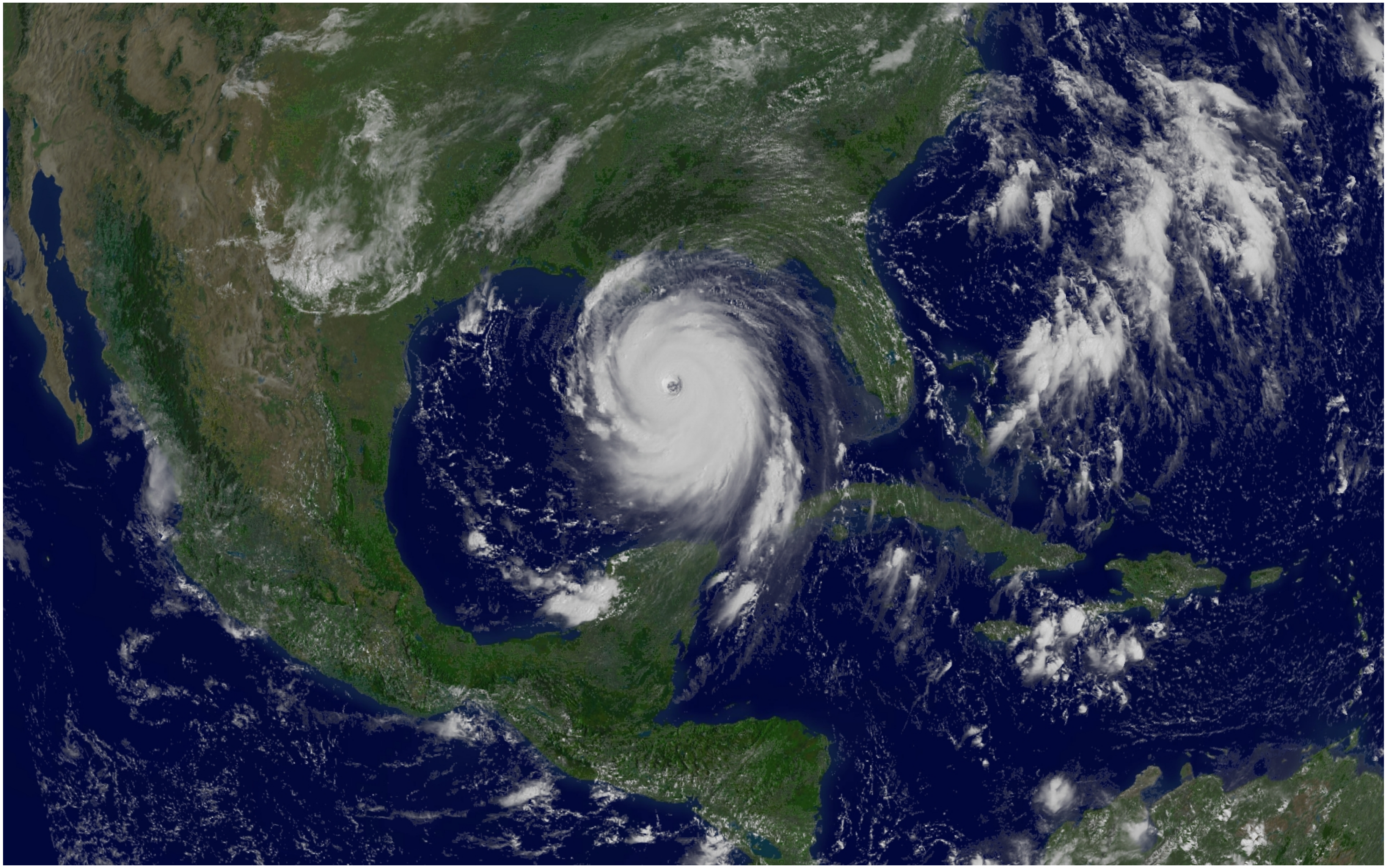
The requirement that the Coriolis force should partially balance the pressure force dictates the direction of airflow around the high/low.

Boys-Ballot Law: Stand with your back to the wind, and the **low** pressure area is on your **left**.



This phenomenon causes **cyclones** (low pressure regions) and their associated weather systems (eg. Hurricanes) to have an **anti-clockwise** rotation in the **northern** hemisphere and a **clockwise** rotation in the **southern** hemisphere.





Hurricane Katrina

Debunking a myth

It is often said that the same effect causes water to go down a plughole anti-clockwise in the northern hemisphere, and clockwise in the southern hemisphere.

This is not true!

Let's make an order of magnitude calculation:

Assume that the water going down the plughole moves at about $1 ms^{-1}$

At 56^o north, coriolis acceleration = $2\,\omega\, v \sin\alpha = 2 \times 7.27\times10^{-5}\, s^{-1} \times 1 ms^{-1} \times \sin 56^o$

$\approx$ **0.0001 ms^{-2}**

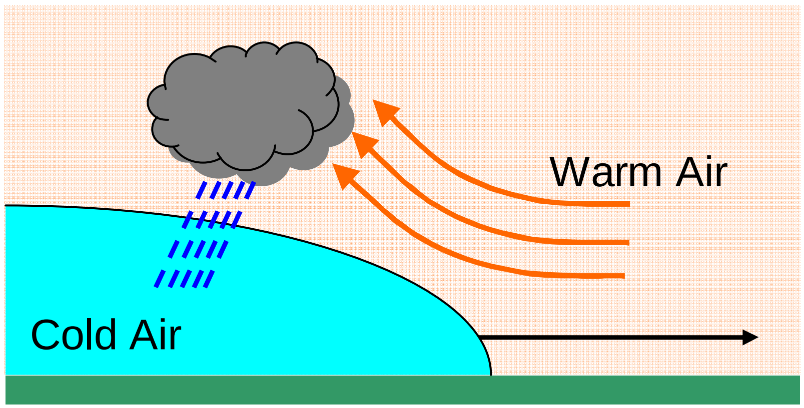
This is tiny and can have no discernable effect on the water. Whether or not the flow moves clockwise or anti-clockwise is due to the geometry of the sink!

Fronts

An **air mass** is a region of air whose properties are constant throughout its entire horizontal extent. This is caused by the air remaining in contact long enough to allow variations in properties to be reduced and reach equilibrium.

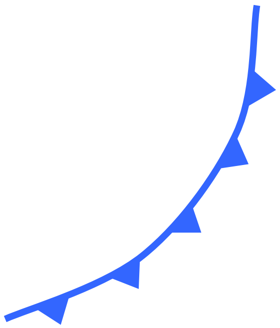
A **front** is the interface between two air masses.

Cold Front

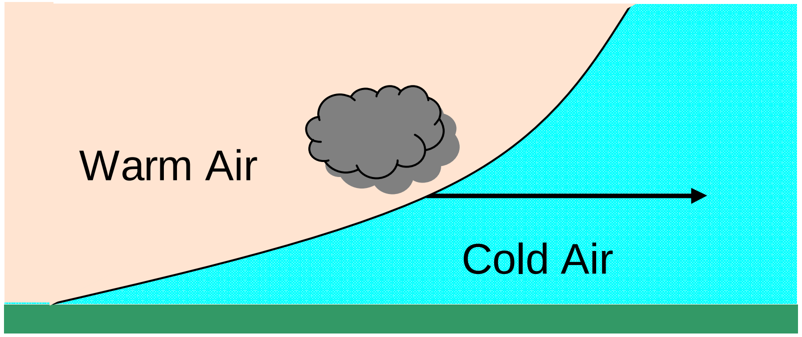


A cold front is where cold air is advancing, pushing warm air ahead of it. The cold air displaces the lighter warm air, pushing it upwards. Expansion (not contact) cools the warm air, and may cause water in the warm air to condense into clouds and potentially rain.

On weather charts, a cold front is symbolically represented by a solid blue line with triangles pointing in the direction of movement.

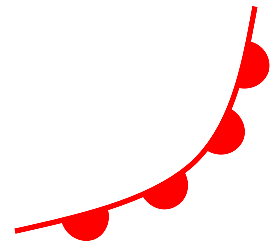


Warm Front

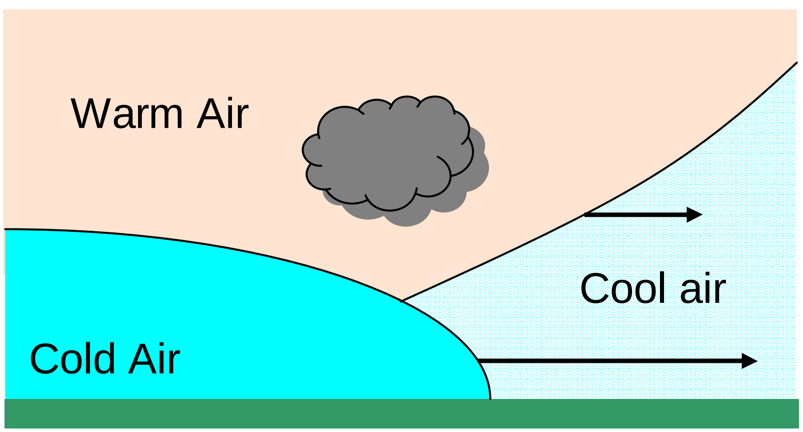


A warm front is where warm air pushes cold air ahead of it. Again the warm air will be pushed up, potentially forming clouds and rain.

A warm front is represented by a red line with semicircles pointing in the direction of motion.

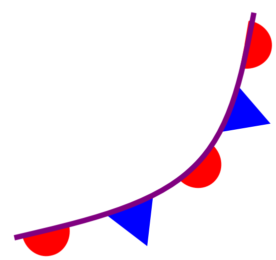


Occluded Front



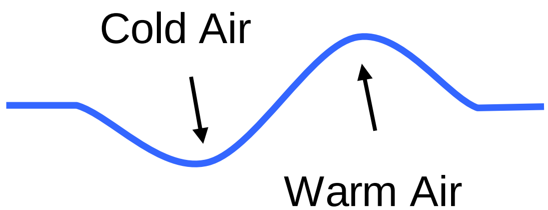
Cold fronts travel more quickly than warm fronts, so a cold front may overtake a warm front. When this happens we have an occluded front. The warm air is undercut and lifted from the ground.

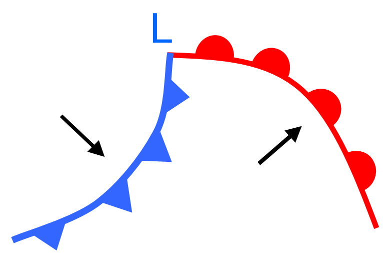
It is represented by a purple line with both semicircles and triangles.

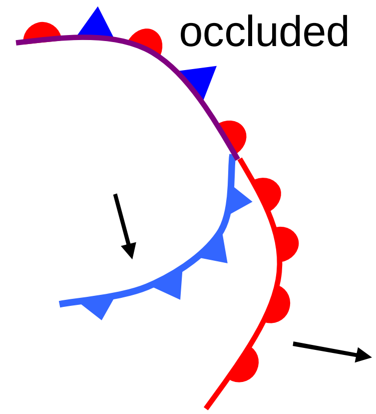


Depressions from the Polar Front

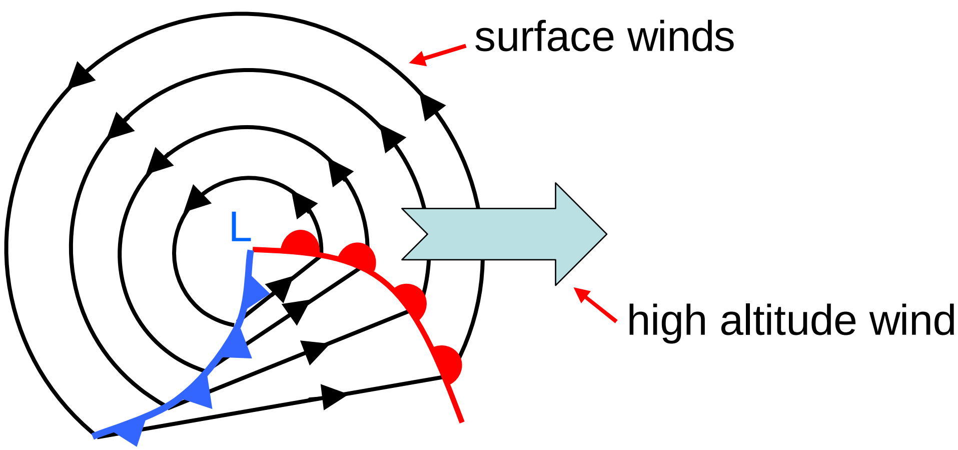
Instabilities in the polar front can easily develop into depressions which dominate the Scottish weather.





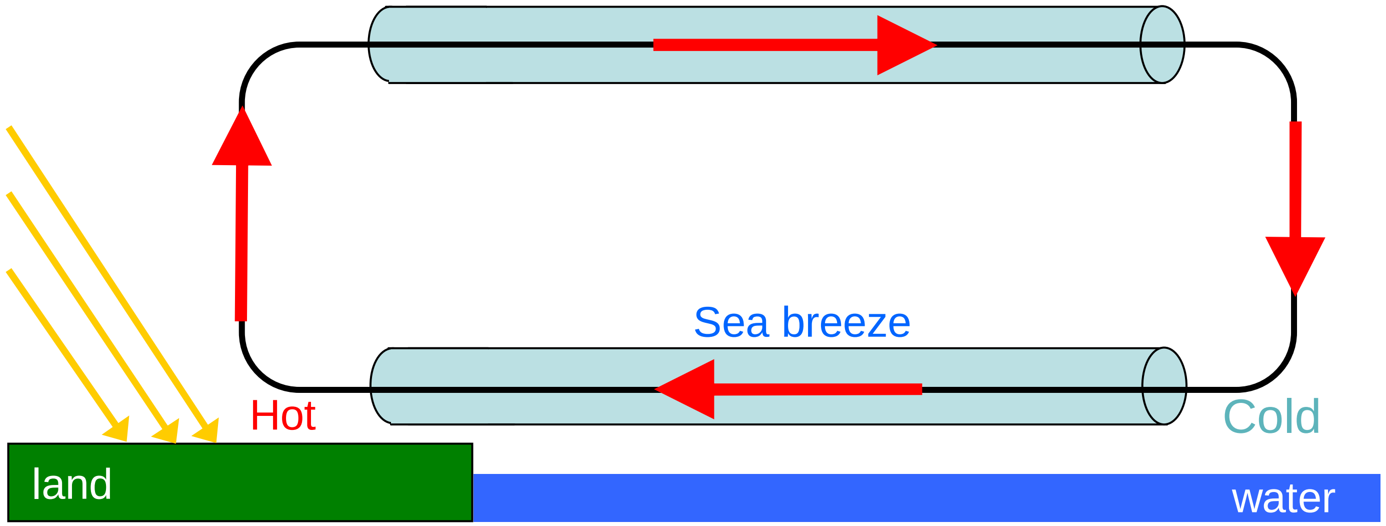


Instability in polar front breaks off and forms a depression.



Sea Breezes

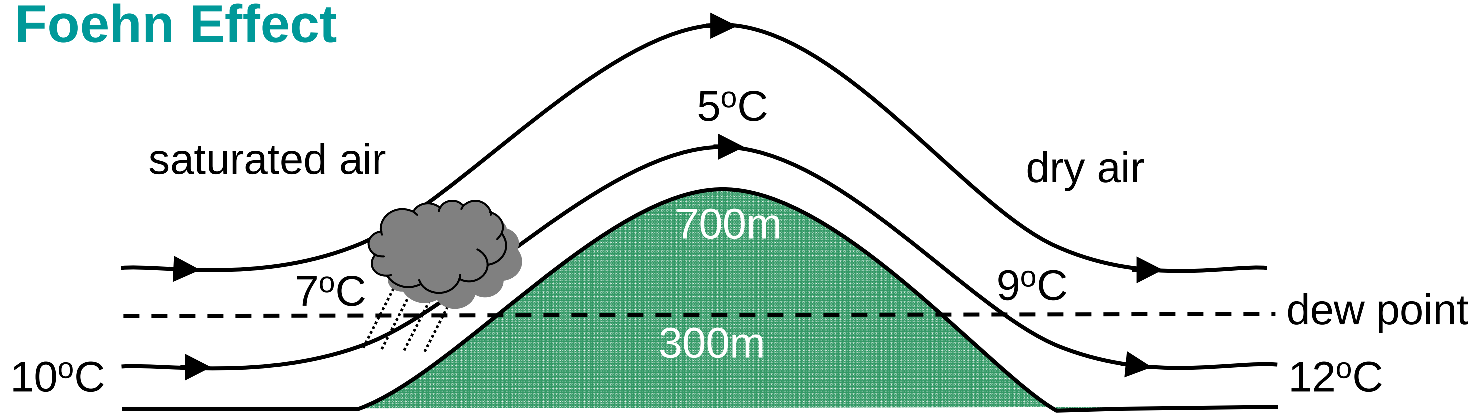
The physics of sea breezes is very similar to that of the Hadley cell. In this case, it is the land which heats up from the sun's rays while the sea remains cool (the oceans have a very high heat capacity).



Valley Winds

Similarly, valley winds are generated in mountain areas such as the alps. High altitude areas become warmer than the lower ends of the valleys and so winds blow up the valleys.

The Foehn Effect



If an air current is forced to pass over a mountain range it may be cooled sufficiently to reach its dew point. Thus the windward side of mountains get a lot of rain.

Since the air has lost its water, when coming down the other side of the mountain it is dry and will heat rapidly (at the DALR value of 10K/km). Thus one often finds a hot dry wind blowing off mountains.

The **Chinook** is a hot dry air on the Eastern slopes of the Rocky mountains.

The **Froehn** is a warm air which blows off the Alps into the valleys to the north.

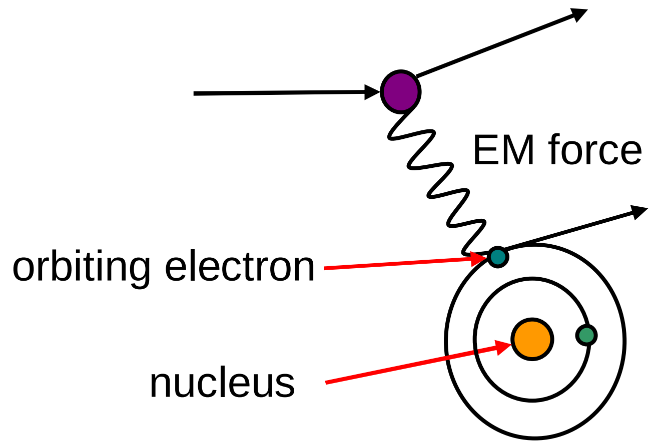
This is why the west of Scotland has a much higher rainfall than the east.

4. Health Physics

This section will discuss the effects of radiation on health. What are the different types of radiation? How harmful are they? How much radiation does a normal person receive? Where do we find sources of radiation?

Ionising Radiation

There are 3 basic typed of ionising radiation:



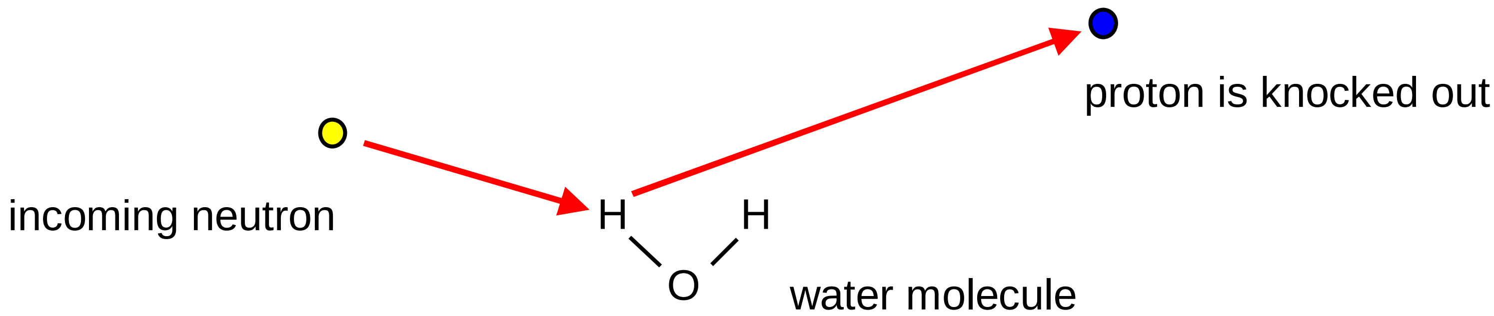
1. Charged particle radiation

   α radiation are Helium nuclei, ${}^{4}_{2}He$
   β radiation are electrons, e^{-}.

   Charged particles interact with the orbiting electrons of atoms as they pass through a material by the **electromagnetic force**. They knock the electron out of orbit creating a positively charged **ion** and a **free electron**. This ionisation process requires energy, which is provided by the kinetic energy of the radiating particle. Consequently the α or β particle loses energy continuously and this radiation is **not very penetrating**. α radiation is stopped by a sheet of paper and β radiation is stopped by about 1cm of human tissue.

2. Neutron radiation

Neutrons are neutral so cannot interact with the electromagnetic force, and cannot use electromagnetism to ionise a material. Nevertheless, if a neutron physically hits a (charged) **proton** in the nucleus of an atom, it can knock it out of the atom and create ions.



The ejected proton is charged so it will in turn ionise more atoms.

Since the human body is mainly water, the above reaction will happen often. Although the loss of a water molecule is no great problem in itself, this process with create chemically reactive **free radicals** such as H^+ and OH^- which diffuse through the cell and cause damage.

Even worse, the neutrons may hit and damage the DNA of a cell, which may alter its behaviour (e.g. cancer) or kill it.

3. X-Rays and γ-radiation

Both X-Rays and γ-radiation (gamma radiation) are made up of photons of different wavelengths.

X-Rays	0.03nm	$\rightarrow$	3nm
γ–rays	0.003nm	$\rightarrow$	0.03nm

The photon only has a very small probability of hitting an orbital electron so these radiations are **very penetrating**.

When a collision does happen, the photon transfers energy to a bound electron and knocks it out of the atom (the photo-electric and Compton effects).

The ejected electron then ionises the material just like β radiation.

Raditaion Dose

How much damage the radiation does to the human body is linked to the amount of ionisation. We would therefore like to express “radiation dose” in terms of the number of ionisations produced per unit mass of tissue irradiated.

The energy required to create an ion pair is independent on the type of incident radiation, so it is more convenient to express radiation dose as the **energy absorbed** by tissue per unit mass.

I Gray = Energy absoption of 1 J/kg

[An older unit, the “rad” is sometimes used: 1 Gray = 100 rad.]

However, the number of ionisations (or energy absorbed) is not the only factor. If the radiation produces a very dense track of ionisation (rather than being more spread out) then it will be more biologically damaging. Each type of radiation is assigned a **Quality Factor**, which takes into account this effect, with higher quality factors causing more damage.

Radiation type	Quality Factor
X-Rays and γ-rays	1
β radiation	1
α radiation	20
Neutron radiation	20

depends on neutron energy

α radiation and neutron radiation are 20 times more damaging than β radiation, X-rays and γ-radiation!

To measure the damaging effects of radiation on the human body we use:

Dose equivalent = Absorbed dose (in Grays) $\times$ Quality factor

this is measured in Sieverts (Sv)

[Again, an older unit, the "rem", is sometimes used, with 1 Sv = 100 rem.]

Health Effects

- Damage to health from radiation depends on whether or not the exposure is of **short duration** (minutes or hours) or **long duration** (weeks). The exposure of the same dose over weeks rather than minutes will be much less damaging because the body has a chance to repair the damage.

Exposure of a large dose in a short time is called **acute exposure**.

Radiation Dose (Sv)	Health effect	
> 0.1	damage to blood chemistry	short term recovery certain
0.5	nausea	short term recovery certain
0.75	vomiting, hair loss	short term recovery certain
1	haemorrhage	short term recovery certain
4	death within 2 months 50% likely	
>6	death certain	

- Long term health effects of radiation of **non-acute exposures** or small acute exposures are **stochastic**. There may be no immediate clinical effect but there is a subsequent probability of developing malignant diseases such as leukaemia or bone cancer.

- Analysis of people who have had a large radiation dose, such as the survivors of Hiroshima and Nagasaki, people living near Chernobyl and hospital patients exposed to radiation as part of their treatment, allows us to estimate the probability of developing a fatal malignancy for a given radiation dose:

  A radiation dose of 1 Sv $\rightarrow$ risk of 1 in 50 (i.e. 2%) of subsequently dying of cancer.

  It is not clear yet whether risk is directly proportional to dose, but this is usually assumed.

- Exposure to radiation can also effect the unborn **next generation**. If one parent-to-be is exposed to a radiation dose of **1 Sv prior to conception**, the risk of a serious birth defect in their children or grandchildren is 4 in 1000 (0.4%).

Exposures to ionising radiation

Radioactivity is a perfectly natural phenomenon and almost everything is radioactive to some extent.

Natural sources of radiation:

[1 Sv = 1000 mSv]

Source	Dose mSv/year
Cosmic radiation - Radiation bombarding Earth from outer space	0.27
Potassium, Uranium and Thorium naturally occurring in the rocks and soil	0.28
The human body contains about 150g of potassium, 0.012% of which is radioactive 40K	0.39
Radon, a radioactive noble gas produced in the decay of radium in the soil	2.0
Total	**3.0**

This varies from place to place. For example, Aberdeen is particularly radioactive!

Other sources of radiation:

[1 Sv = 10^6 μSv]

Source	**μSv**
Radiation received by bone marrow from a chest X-Ray	100
Annual dose from consumer products (e.g. fluorescent watches)	100
Average airline passenger (10 flights/year)	30
Average dose from an old TV set per year	2-15
Average annual dose (for US citizen) from nuclear power plants	0.2

Adding up natural and unnatural sources of radiation, the average person is exposed to about 3.6 μSv/year.

Legal restrictions on radiation exposure (Ionising Radiation Regulations Act 1999):

Radiation worker	20 mSv/year or 100mSv in 5 years with a maximum of 50mSv/year
Member of the general public	1mSv/year