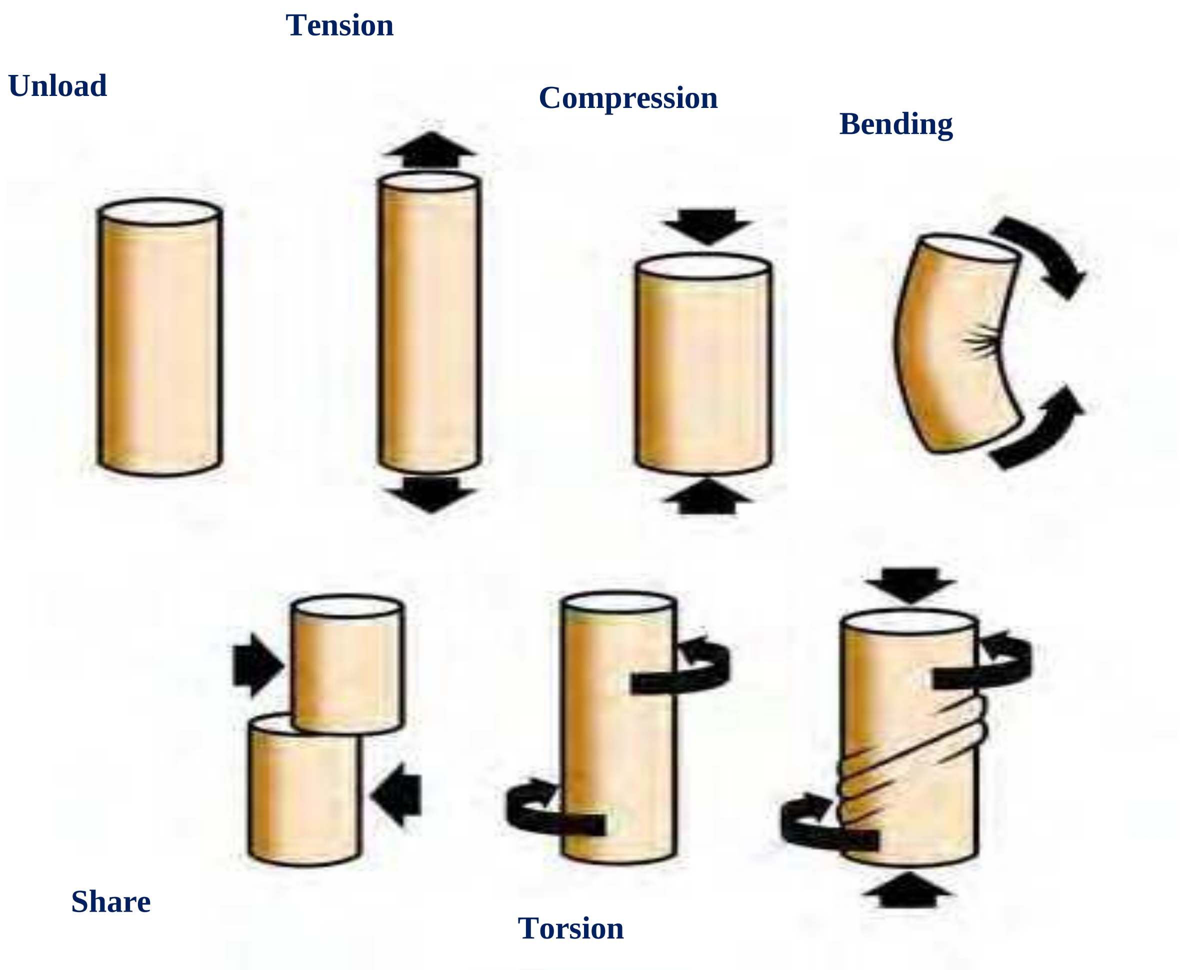
Mechanical properties of biological tissues (MPBT)

I. Theoretical Background

1. Types of deformations:



- **Tensile deformation –** tension and compression of a sample

- **Shear deformation -** is the deformation of the body when it's flat layers are displaced relatively each other.

- **Bending deformation -** appears in a sample subjected to an external load applied perpendicularly to a longitudinal axis of the element.

- **Torsional deformation** – appears in a sample when one of its sections is immobile whereas the other is influenced a pair of forces, the moment of which is directed along the sample's axe

2. Physical quantities which are used to characterize Mechanical properties of materials:

$$\textbf{Strain}\ \varepsilon = \frac{\Delta l}{l_\circ}$$

– the ratio of absolute deformation to initial size value

$$\textbf{Stress}\ \sigma = \frac{F}{S}$$

– internal forces acting on the object as a reaction to the external applied forces (per unit of area)

$$\textbf{Stiffness}\ k = E\cdot \frac{S}{l_\circ}$$

E – is Young's Modulus – characteristic of elastic properties (stiffness) of material.

Young's Modulus does not depend upon the size or shape of the object, but only the material the object is composed of.

From the **Hooke's law**:

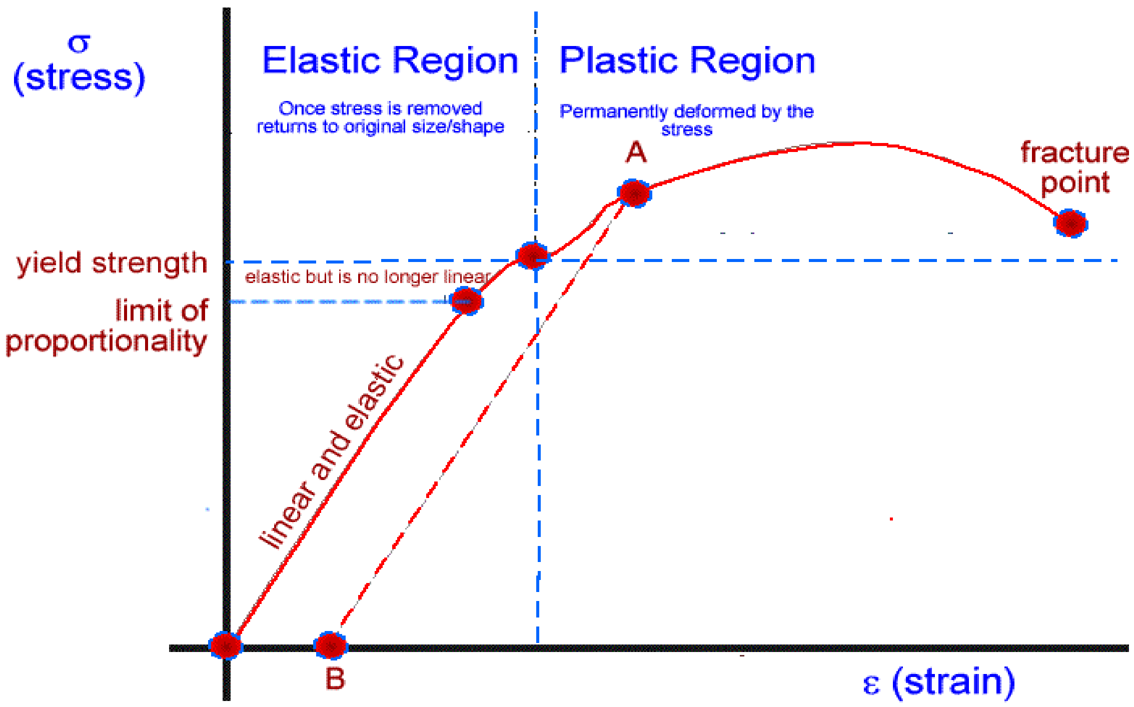
$$F = k\cdot x = k\cdot \Delta l = E\cdot \frac{S}{l_\circ}\Delta l$$

or

$$\frac{F}{S} = E\cdot \frac{\Delta l}{l_\circ} \quad or \quad \sigma = E\cdot \varepsilon$$

Relationship between stress and strain can be represented graphically and is called ***stress – strain diagram*** which is a characteristic of an elastic properties of a material.

3. Stress – strain diagram and an elastic properties of a materials:



- **Elasticity** is the capability of a strained body to recover its size and shape after external forces are removed.

  - the property by virtue of which a body tends to regain its original shape and size when external forces are removed. If a body completely recovers its original shape and size, it is called perfectly elastic.

- **Resilience** is the ability of a material to absorb energy when it is deformed elastically (ability to renew the size and shape after the stress is relieved)
- the area under the linear portion of a stress-strain curve is the resilience of the material

- **Stiffness** is the rigidity of an object — the extent to which it resists deformation in response to an applied force

- **Plasticity** describes the deformation of a material undergoing non-reversible changes of shape in response to applied forces
- the transition from elastic behavior to plastic behavior is called ***yield***

- **Fragility** is a property of material to be subjected to destruction

- the transition from plastic behavior to fracture point

- **Viscosity** is a property of a material to resist changing of its form under the shear deformation

- **Fluidity** is ability of a material (substance) to flow, the physical characteristic which is reciprocal to viscosity

4. Types of biological tissues

Epithelial Tissue	**The skin, outer and inner covering of the body and the bodies organs and cavities**		
Connective Tissue = Extracellular Matrix + Some types of specific cells: fibroblasts, adipocytes, macrophages, mast cells, leukocytes.	**Loose connective tissue**		
	Fibrous (Dense) connective tissue		
	Adipose (Fat) tissue		
	Curtilage		
	Bone tissue		
	Blood		
Muscle Tissue	**Skeletal muscles**	**Smooth muscles**	**Cardiac muscles**
Nerve Tissue	**Neurons**		

Video: types of tissues

5. Elastic Properties of Biological Tissues

The elastic properties of biological tissues basically depend on:

- the structure of tissue – components compound
- the functional (actual) state of tissue which tends to change with age
- the applied load: type and magnitude
- the time (duration), direction and velocity of an applied load

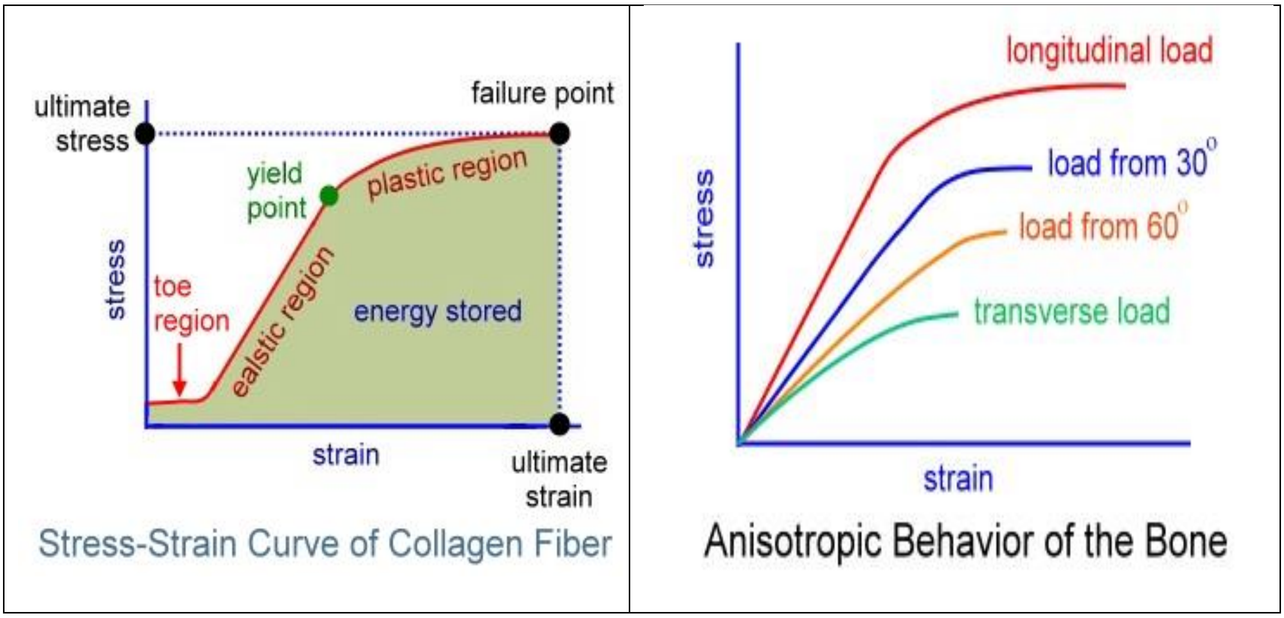
1. Most biological tissues are composite materials and possess both fluid and solid properties. From mechanical point of view it means that biological tissues are viscoelastic.

2. There are two basic protein components of biological tissues which sufficiently influence the overall mechanical properties and behavior of tissues in which they appear: **collagen** and **elastin**.

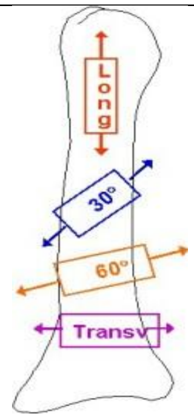
3. **Collagen** is a protein made of crimped fibrils that aggregate into fibers. The mechanical properties of collagen fibrils are such that each fibril can be considered a mechanical spring and each fiber as an assemblage of springs.

    - Like a mechanical spring, the energy supplied to stretch the fiber is stored and it is the release of this energy that returns the fiber to its unstretched configuration when the applied load is removed.
    - Collagen fibers possess a two-phase, solid–fluid, or viscoelastic material behavior with a relatively high tensile strength and poor resistance to compression.

Stress–strain diagram for collagen and bone



Failure of bone may occur when the applied stresses exceed the ultimate strength limit, which may result from excessive stresses, or weak material, or both.



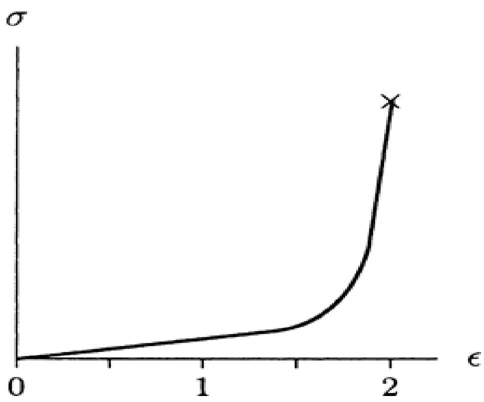
Fracture under the stretching $\boldsymbol{\sigma} = \mathbf{1 \cdot 10^9 \frac{N}{m^2}}$ $\quad \boldsymbol{\varepsilon} = \mathbf{0.05}$

Fracture under the compression $\boldsymbol{\sigma} = \mathbf{2 \cdot 10^9 \frac{N}{m^2}}$ $\quad \boldsymbol{\varepsilon} = \mathbf{0.1}$

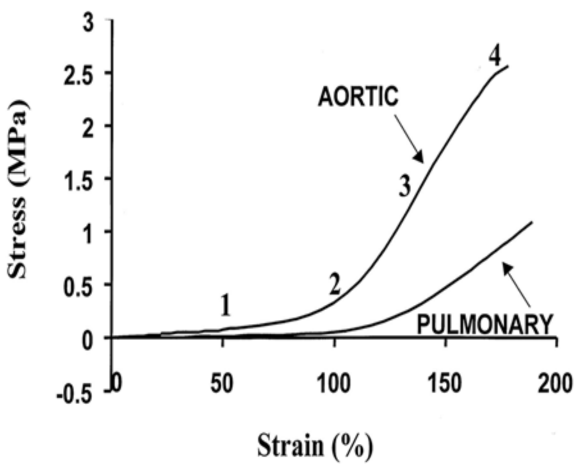
4. **Elastin** is fibrous protein with material properties that resemble the properties of rubber.
    - Elastin and microfibrils form elastic fibers that are highly extensible, and their extension is reversible even at high strains.
    - Elastin fibers behave elastically with low stiffness up to about 200% elongation followed by a short region where the stiffness increases sharply until failure.

Elastin fibers possess a low-modulus elastic material property, while collagen fibers show a higher modulus viscoelastic material behavior.

Stress–strain diagram for elastin and arterial blood vessels

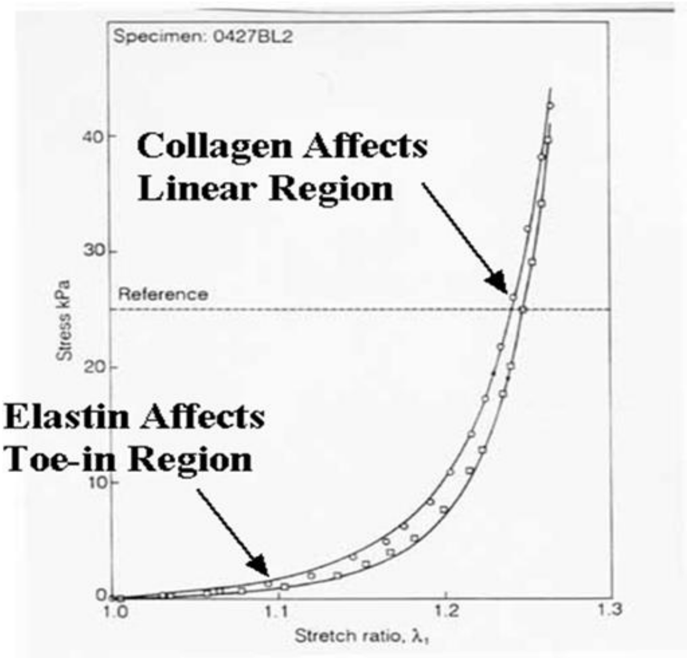


Elastin



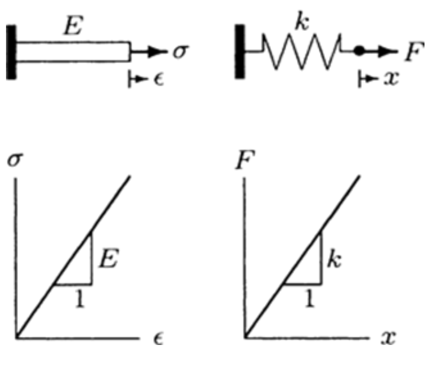
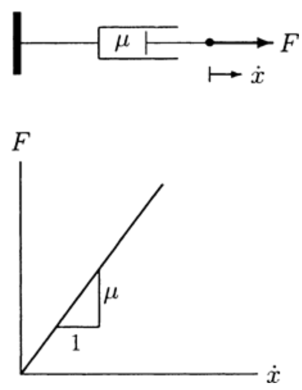
Aorta

Elastin fibers behave elastically with low stiffness up to about 200% elongation followed by a short region where the stiffness increases sharply until failure.



6. Empirical Models of Viscoelasticity

1. Models of Viscoelasticity can be built on the base of two simple models: the Model of Elasticity – “Spring” and the Model of Viscosity – “Dashpot”

Model of Elasticity - Spring	Model of Viscosity - Dashpot
$\sigma = E\epsilon$ $F = kx$ 	$F = bv$ $\sigma = \eta\, \varepsilon'$ 

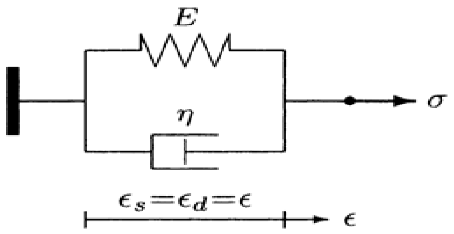
Spring represents elastic and dashpot represents viscous material behaviors

It is assumed that:

- a constantly applied force (stress) produces a constant deformation (strain) in a spring and a constant rate of deformation (strain rate) in a dashpot.
- The deformation in a spring is completely recoverable upon release of applied forces, whereas the deformation that the dashpot undergoes is permanent.

2. Kelvin–Voight Model

The Kelvin–Voight model is a system consisting of a spring and a dashpot *connected in a parallel arrangement.*



A stress $\boldsymbol{\sigma}$ applied to the entire system will produce stresses $\boldsymbol{\sigma}_s$ and $\boldsymbol{\sigma}_d$ in the spring and the dashpot. The total stress applied to the system will be shared by the spring and the dashpot such that:

$$\boldsymbol{\sigma = \sigma_s + \sigma_d}$$

As the stress s is applied, the spring and dashpot will deform by an equal amount because of their parallel arrangement.

Therefore, the strain $\boldsymbol{\varepsilon}$ of the system will be equal to the strains $\boldsymbol{\varepsilon_s}$ and $\boldsymbol{\varepsilon_d}$ occurring in the spring and the dashpot:

$$\boldsymbol{\varepsilon = \varepsilon_s = \varepsilon_d}$$

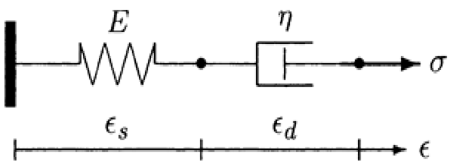
Therefore:

$$\boldsymbol{\sigma = E\cdot \varepsilon + \eta\cdot\varepsilon'}$$

This is a first order, linear ordinary differential equation. For a given stress $\boldsymbol{\sigma}$, the equation can be solved for the corresponding strain $\boldsymbol{\varepsilon}$. For prescribed strain $\boldsymbol{\varepsilon}$, it can be solved for stress $\boldsymbol{\sigma}$.

3. Maxwell Model

The Maxwell model is constructed by connecting a spring and a dashpot in a series.



In this case, a stress $\boldsymbol{\sigma}$ applied to the entire system is applied equally on the spring and the dashpot

$$\boldsymbol{\sigma = \sigma_s = \sigma_d}$$

and the resulting strain $\boldsymbol{\varepsilon}$ is the sum of the strains in the spring and the dashpot

$$\boldsymbol{\varepsilon = \varepsilon_s + \varepsilon_d}$$

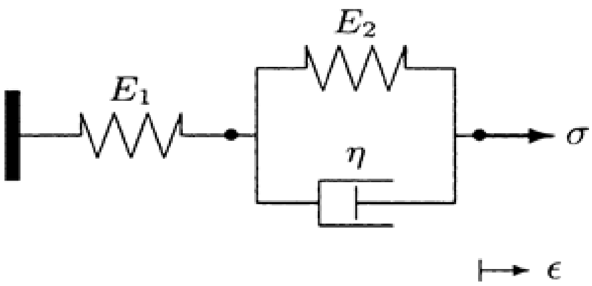
Therefore:

$$\boldsymbol{\eta \cdot \sigma' + E \cdot \sigma = E \cdot \eta \cdot \varepsilon'}$$

A viscoelastic Maxwell material model is commonly used to represent the viscoelastic properties of blood.

4. Standard Solid Model

The standard solid model is composed of a spring and a Kelvin–Voight model connected in a series.



In this case:

$$\boldsymbol{\varepsilon = \varepsilon_1 + \varepsilon_2 \qquad \sigma = \sigma_1 = \sigma_2}$$

$$\boldsymbol{\sigma_1 = E_1 \cdot \varepsilon_1 \qquad \sigma_2 = E_2 \cdot \varepsilon_2 + \eta \cdot \varepsilon_2'}$$

The resulting differential equation is:

$$\boldsymbol{(E_1 + E_2) \cdot \sigma + \eta \cdot \sigma' = E_1 \cdot E_2\, \varepsilon + E_1 \cdot \eta \cdot \varepsilon'}$$

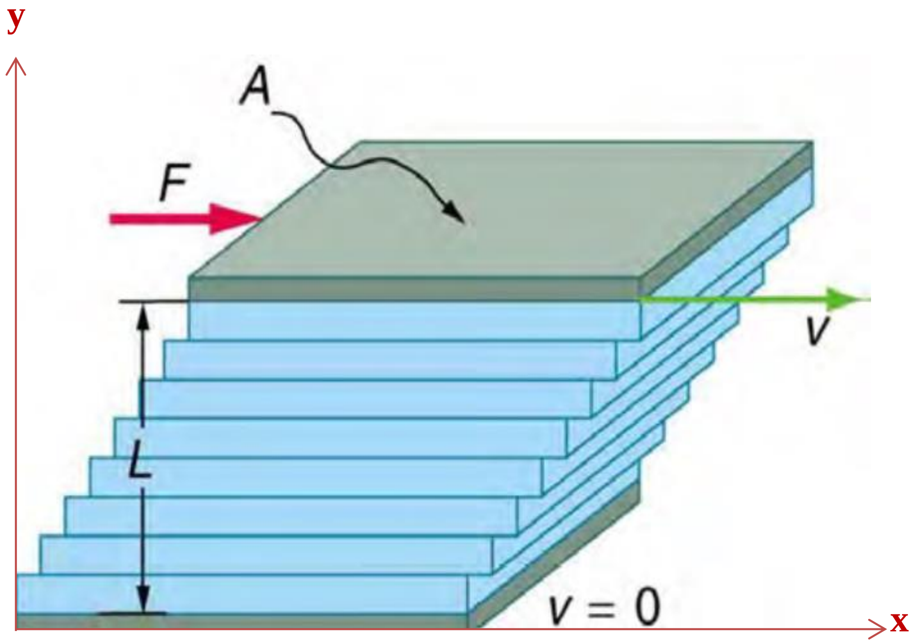
The standard solid model is a three-parameter (E_1, E_2, η) model and is used to describe the viscoelastic behavior of a number of biological materials such as the cartilage and the white blood cell membrane.

7. Viscosity of biological fluids

1. The viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress. Shear stresses applied to an element of fluid cause a relative movement of the surfaces (layers).

The viscosity of a fluid is determined by how molecules constituting the system interact.

A fluid is thought to be ideal if it has zero viscosity.



Each successive layer from the top down exerts a force upon the one below it, trying to drag it along, producing a continuous variation in speed from *v* to 0.

Friction between the fluid and the moving boundaries/plates causes the fluid to shear. The force required for this action is a measure of the fluid's viscosity.

$$\boldsymbol{F = \eta \cdot A \cdot \frac{\partial V}{\partial y}} \textbf{ or } \boldsymbol{(shear\ stress)\ \tau = \frac{F}{A} = \eta \cdot \frac{\partial V}{\partial y}}$$

F is directly proportional to:

1) the area ***A*** of the adjacent layers

2) the strain (shear) rate or the rate of shear deformation which is the gradient of velocity of layers in the direction perpendicular to the layers

3) the coefficient of internal friction or **viscosity** $\boldsymbol{\eta}$

The SI units of viscosity are: $[\eta\ ] = \frac{N\cdot s}{m^2} = Pa\cdot s$

Viscosity depends on nature of fluids, external conditions (temperature, pressure (for gases)) and type of the flow.

2. There exist two types of fluids with respect to viscosity behavior: Newtonian fluids and non-Newtonian fluids.

A fluid is thought to be Newtonian if it has a viscosity which is independent of shear rate (applied force or pressure), otherwise the fluid is considered to be a non-Newtonian.

For Newtonian fluids ***viscosity*** depends only on temperafure and nature of the fluid.

Data of viscosity (in box) for some important biofluids in millipascal (mPa) with respect to temperature in Celsius (t° C)

Temperature t° C	**0**	**20**	**37**	**40**	**100**
fluid					
Water	**1.792**	**1.002**	**0.6947**	**0.653**	**0.282**
Blood plasma		**1.810**	**1.257**		
Whole blood		**3.015**	**2.084**		
Ethyl alcohol		**1.20**			
Methanol		**0.584**			
Glycerin		**1500**			
Oil (olive)		**138**			
Milk		**3.0**			
Honey		**2000 - 10000**			

3. There are two types of fluids flow related to viscosity: the laminar flow and the turbulent flow.

- **Laminar flow is characterized by the smooth flow of the fluid in layers that do not mix.**
- **Turbulent flow, or turbulence, is characterized by eddies and swirls that mix layers of fluid together.**

There is a criteria (indicator) called the **Reynolds number Re** which can reveal whether flow is laminar or turbulent.

For flow in a tube of uniform diameter, the Reynolds number is defined as:

$$\mathbf{Re} = \frac{\rho v d}{\eta}$$

where :

ρ - is the density of the fluid; v - is the speed of the fluid; d – is the diameter of the tube;

η – is the viscosity of the fluid

The Reynolds number is a unitless quantity. Experiments have revealed that **Re** is related to the onset of turbulence.

The relationship between the Reynolds number and the characteristics of the flow

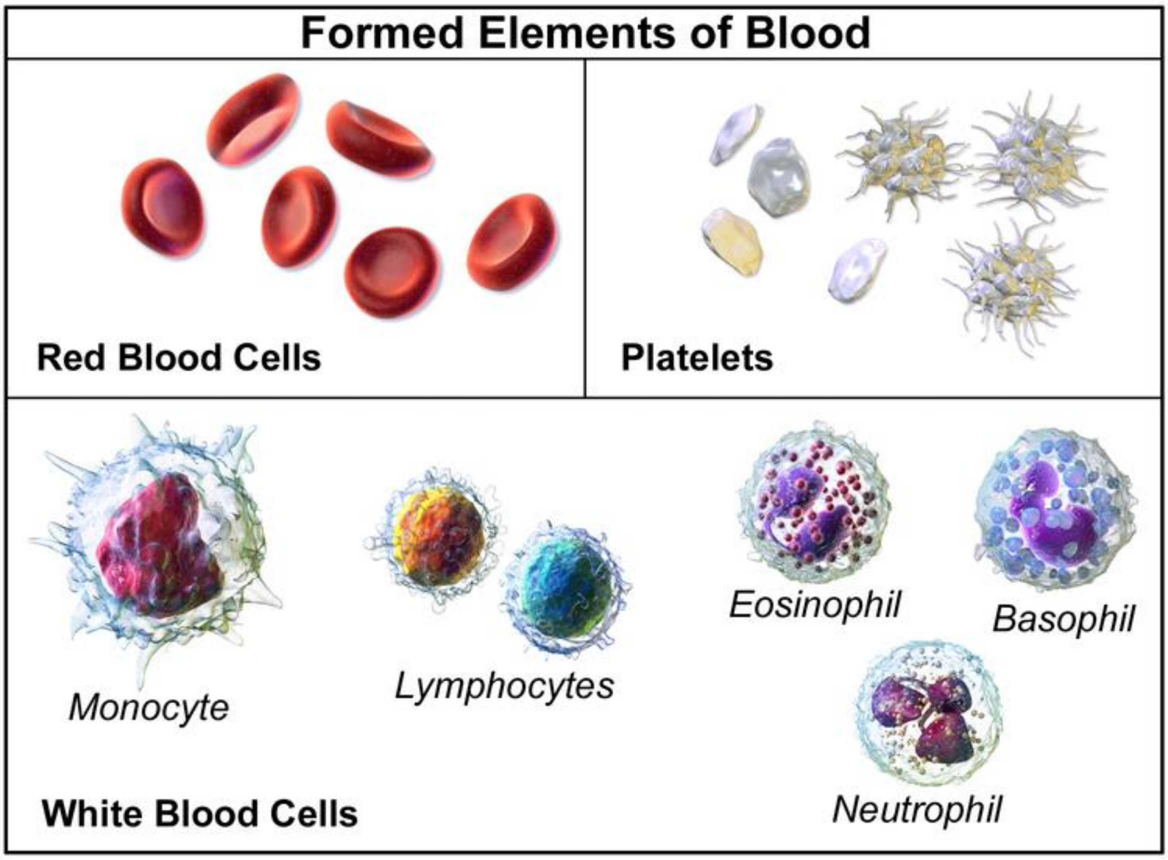
$0 < \mathrm{Re} \leq 1$	Flow is laminar in form of ‘creeping ‘flow, entirely dominated by viscosity.
$1 < \mathrm{Re} \leq 2000$	Laminar flow, determined by fluid viscosity and kinetic energy of flow.
$2000 < \mathrm{Re} \leq 3000$	Flow is unstable – that is, it can be laminar, but small obstructions and surface roughness can make it turbulent, and it may oscillate randomly between being laminar and turbulent.
$3000 < \mathrm{Re} \leq 10000$	Transition to turbulence
$10\,000 < \mathrm{Re}$	Flow is turbulent relatively determined by laminar viscosity

The blood flow through most of the body is a quiet, laminar flow. The exception is in the aorta, where the speed of the blood flow rises above a critical value of 35 sm/s and becomes turbulent.

4. Blood viscosity

Blood is non-Newtonian fluid. This means that blood viscosity is depend of shear rate (applied force or pressure), and actually is affected by blood velocity in vessels.

It happens because blood is not a homogeneous fluid, but consists of a suspension of cells and platelets in a fluid solution called plasma with about equal proportions of each.



- The erythrocytes (Red Blood Cells – RBC) are biconcave discs, essentially toroidal in shape but with the middle filled in. The outer diameter of the torus as 7.5 μm ($10^{-6} m$) and the ring diameter, or cell thickness, as 2 μm. Although these cells are deformable, and are capable of travelling along tubes smaller than their own diameter, the diameter of the red blood cells is about that of the capillaries in the cardiovascular circulation. Adults have about 5 million red blood cells per cubic millimeter of blood.
- The leukocytes (White Blood Cells – WBC) are spherical with diameters of about 7 μm, and their major role is in disease-defense mechanisms. Leukocytes make up about 0.3% of the blood volume.

- The platelets are fragments of very large parent cells (megakaryocytes), and are much smaller than the red or white cells. They play a key role in clotting processes. The platelets make up about 0.15% of the blood volume.
- The plasma (colloidal solution with a total concentration of protein of 6-9%) is about 90% water and in isolation behaves as a Newtonian fluid, with a viscosity of 0.0012 Pa·s.

The percentage of the volume occupied by the red cells is called the *hematocrit* (the ratio of the volume of red blood cells to the volume of plasma.

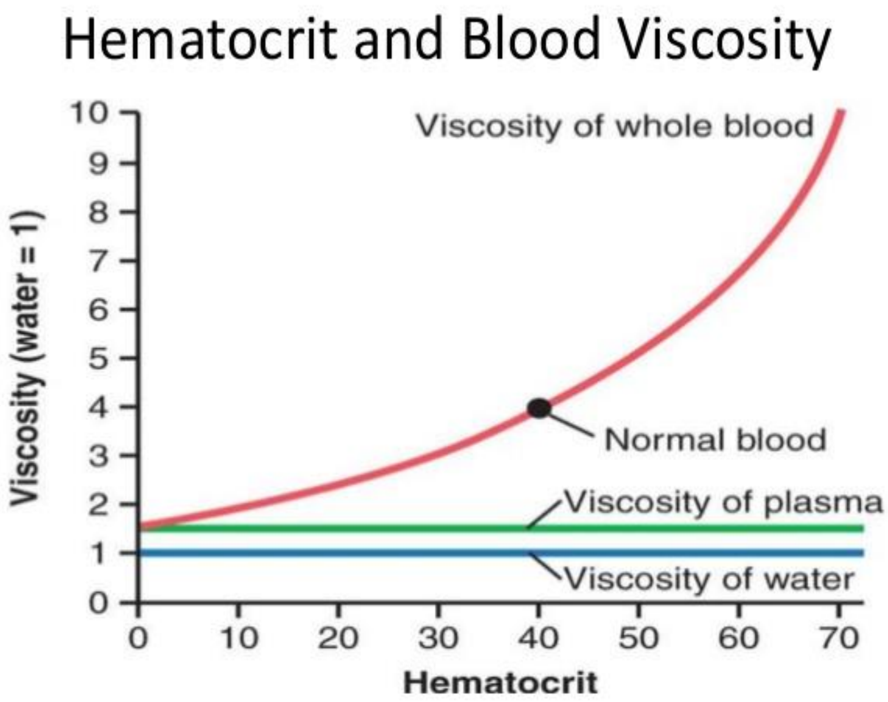
$$\text{Hct} = \frac{V_{formed\ elements}}{V_{plasma}} \approx \frac{V_{erythrocytes}}{V_{plasma}}$$

A typical value of the hematocrit in a healthy individual is about 45%.

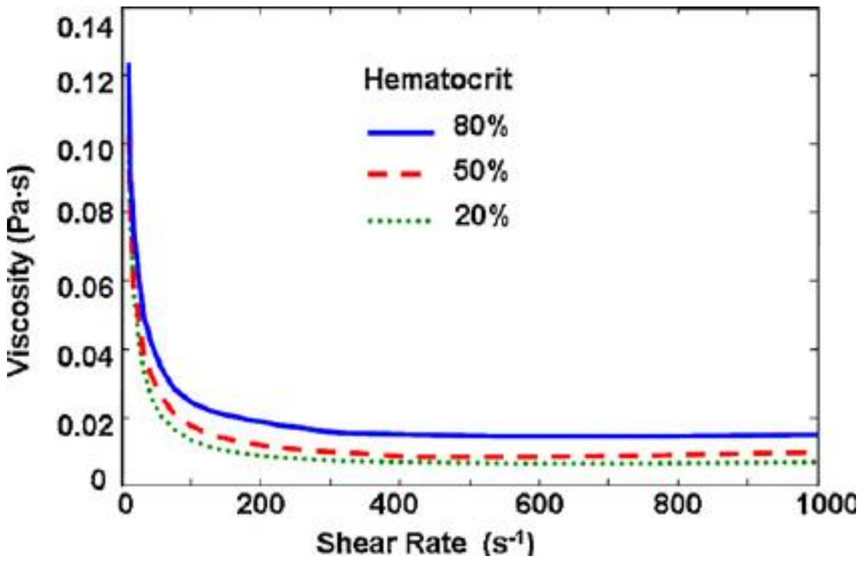
The bulk viscosity of the blood is a function of the hematocrit and increases as the hematocrit increases:

$$\eta = \eta_0 \cdot (1 + \alpha Hct)^{\beta} \text{ or } \eta = \eta_0 \cdot e^{\gamma \cdot Hct}$$

where: η - blood viscosity, η_0 – viscosity of plasma, α, β, γ – are empirical coefficients.



The blood viscosity reduces as the shear rate increases: At high shear rates (high velocity gradients) in arterial vessels blood viscosity, is close to water viscosity whereas at low rates of shear deformation viscosity five times greater compared with water viscosity.

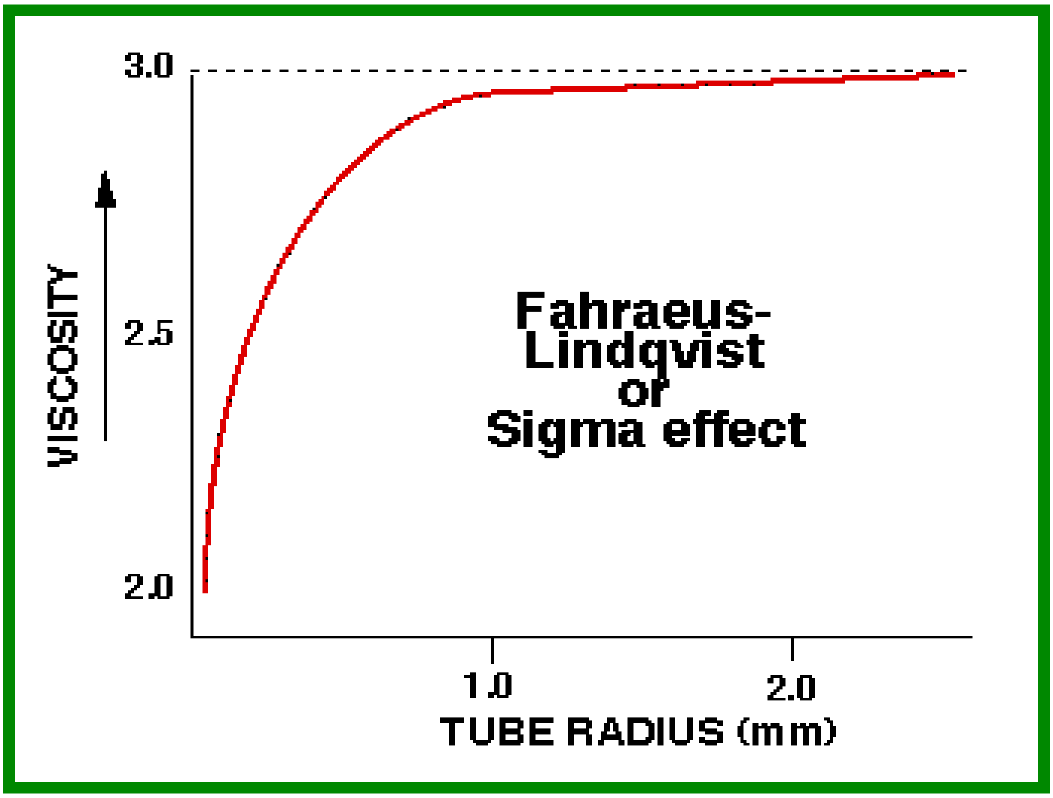


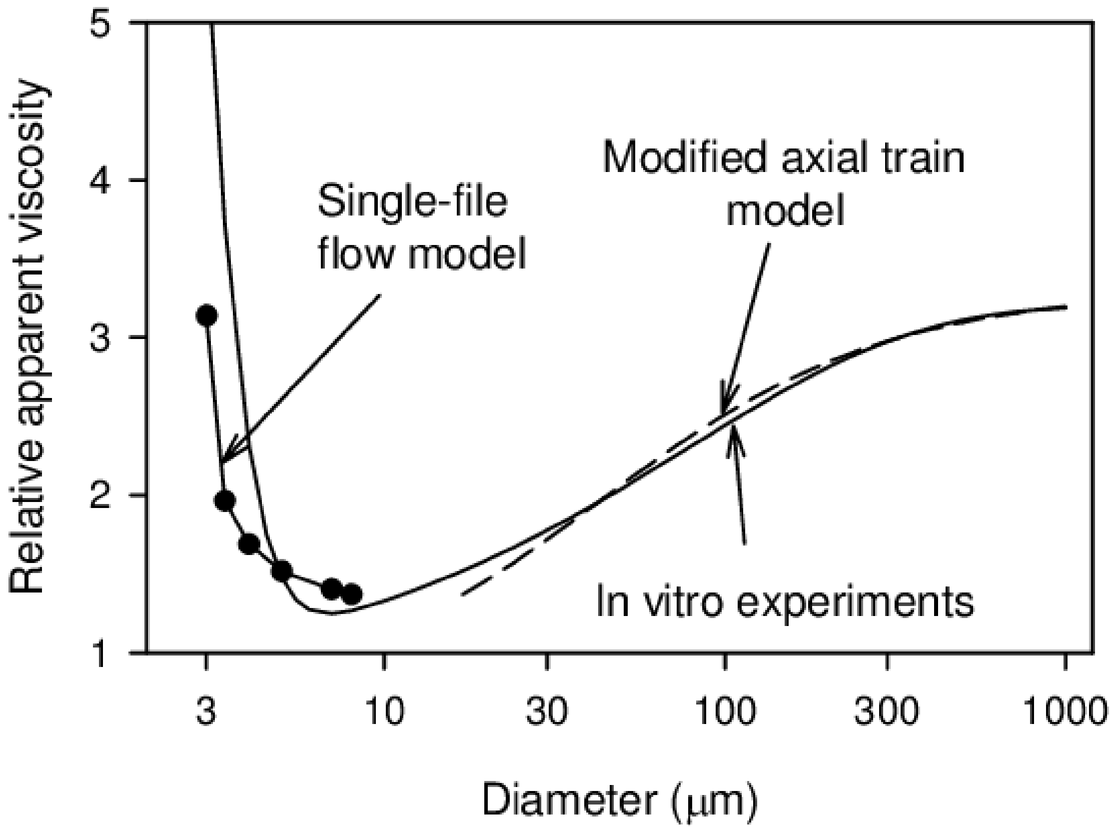
At higher shear rates the viscosity is relatively constant, and for the purposes of analysis of flow in larger vessels at high shear rates blood is often modeled as a Newtonian fluid with a mean viscosity of 0.004 or 0.005 Pa·s.

The viscosity of blood has a primary influence on flow in the larger arteries, while the elasticity, which resides in the elastic deformability of red blood cells, has primary influence in the arterioles and the capillaries.

The Fåhræus–Lindqvist effect /fɑːˈreɪ.əs ˈlɪndkvɪst/ is an effect where the viscosity of blood, changes with the diameter of the vessel it travels through; in particular there's a decrease of viscosity as the tube's diameter decreases (only if the vessel diameter is between 10 and 300 micrometers). This is because erythrocytes move over the center of the vessel, leaving plasma at the wall of the vessel.

The Fåhræus effect is the decrease in average concentration of red blood cells (RBC) in human blood as the diameter of the glass tube in which it is flowing decreases: in blood vessels with diameters less than 500 micrometers, the hematocrit decreases with decreasing capillary diameter**.** The Fahraeus effect occurs because the average RBC velocity is higher than the average plasma velocity.


3.0
2.5
2.0
VISCOSITY
Fahraeus-
Lindqvist
or
Sigma effect
1.0
2.0
TUBE RADIUS (mm)


5
4
3
2
1
Relative apparent viscosity
Single-file
flow model
Modified axial train
model
In vitro experiments
3
10
30
100
300
1000
Diameter (μm)

II. Test Problems

1. Find the elastic deformation energy of a steel rod of mass m = 3.1 kg stretched to a tensile strain $\varepsilon = 1.0 \cdot 10^{-3}$

We assume that the deformation is wholly due to external load, neglecting the effect of the weight of the rod (see next problem). Then a well known formula says,

elastic energy per unit volume

$$= \frac{1}{2} \text{stress} \times \text{strain} = \frac{1}{2} \sigma \varepsilon$$

This gives $\frac{1}{2} \frac{m}{\rho} E \varepsilon^2 \approx 0{\cdot}04$ **kJ for the total deformation energy.**

2. The stress-strain relationships of skeletal muscle can be described by Kelvin–Voigt model. Find the strain of the muscle for 3-minites load of 6.3 N. Young's Modulus E is $1.2 \cdot 10^3$ Pa, cross-section area of the muscle $S = 0.8 \cdot 10^{-6}\ m^2$, muscle viscosity η = 0.125 Pa·s. (6.5)

3. A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $Fs = 6\pi r\eta v$. Show that the terminal speed is given by $v = 2R^2\ g\ \ (\rho s - \rho 1)\ /\ 9\eta$ where R is the radius of the sphere, ρs is its density, and $\rho 1$ is the density of the fluid and η, the coefficient of viscosity.

4. Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.

5. The erythrocyte sedimentation rate (ESR) is the rate at which red blood cells sediment in a period of one hour. Find the change of ESR in the case of 1.5 times increase of erythrocytes diameter. (2.25)

6. When a sphere of radius r_1= 1.2 mm moves in glycerin, the laminar flow is observed if the velocity of the sphere does not exceed v_1= 23 cm/s. At what minimum velocity v_2 of a sphere of radius r_2 = 5.5 cm will the flow in water become turbulent? The viscosities of glycerin and water are equal to η_1 = 1500 Pa·s and η_2 = 1.002 Pa·s respectively.

We know that Reynold's number for turbulent flow is greater than that on laminar flow.

Now, $(R_e)_l = \frac{\rho v d}{\eta} = \frac{2\rho_1 v_1 r_1}{\eta_1}$ **and** $(R_e)_t = \frac{2\rho_2 v_2 r_2}{\eta}$

But, $(R_e)_t \geq (R_e)_l$

so $v_{2_{min}} = \frac{\rho_1 v_1 r_1 \eta_2}{\rho_2 r_2 \eta_1} = 5\ \mu$ **m/s on putting the values.**

7. A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of $5.50 \times 10{-}4$ N is required to glide one over the other at a speed of 1.00 cm/s when their contact area is 6.00 cm2 . What is the oil's viscosity? What type of oil might it be?.

Reading:

- Medical and Biological Physics. Textbook for students of higher medical education institutions / V.I. Fediv, O.I. Olar, V.V. Kulchynskyi, H. Y. Rudko. – Chernivtsi: BSMU Publishing House, 2017. - 342 p. – English.

- Irving P. Herman. Physics of the Human Body. / Springer – Verlag. Berlin, Heidelberg 2007.

Watching:

https://www.youtube.com/watch?v=VvDJyhYSJv8