Fundamentals of Hemodynamics

I. Theoretical Background

1. Principles of Hydrostatics

1.1 Hydrostatic pressure is the weight of the fluid *mg* per unit of the area *S* supporting it (the area of the bottom of the vessel):

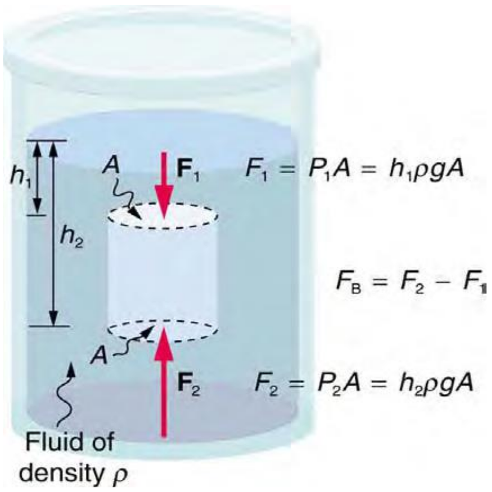
$$\mathbf{P} = \frac{mg}{S} = \frac{\rho g V}{S} = \rho g h$$ - hydrostatic pressure due to the weight of a fluid

1.2 Pascal's Principle:

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of the vessel.

1.3 Archimedes' principle:

The buoyant force on an object equals the weight of the fluid it displaces.



Pressure due to the weight of a fluid increases with depth since $P = h$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force **F**b . (Horizontal forces cancel.)

2. Dynamics of an ideal fluid

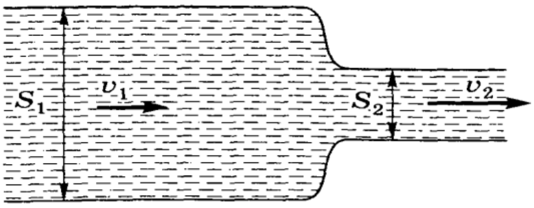
2.1 Flow rate *Q* is defined to be the volume of fluid passing through cross-sectional area of vessel per unit of time:

$$Q = \frac{dV}{dt} = \frac{Sdl}{dt} = S \cdot v$$

2. 2 The equation of continuity for the steady laminar flow:

$$\frac{dm}{dt} = \rho \cdot v \cdot S = \text{const}$$

For incompressible fluid when density $\rho = const$ then



$$v \cdot S = \text{const} \quad \text{or} \quad S_1 v_1 = S_2 v_2$$

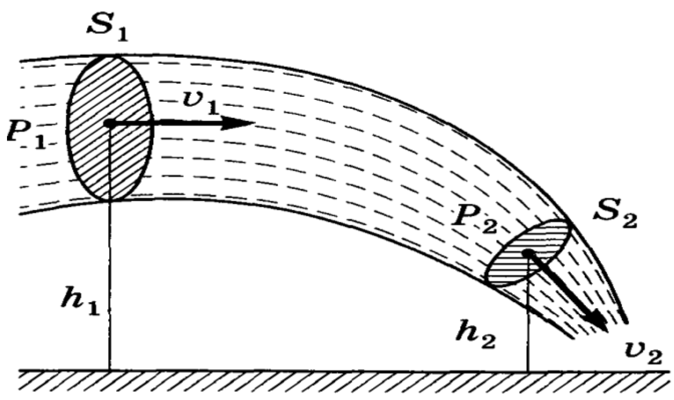
When a flow splits into two currents the sum of flow rates remains constant $Q = Q_1 + Q_2$

2.3 Bernoulli's equation.

When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position.

$$\frac{\Delta m v_2^2}{2} - \frac{\Delta m v_1^2}{2} = W_{net} + (\Delta m g h_1 - \Delta m g h_2)$$

$$W_{net} = (P_1 - P_2)\, S \cdot \Delta l = (P_1 - P_2) \cdot \Delta V$$



Формулу в белом боксе надо убрать.

$$P_2\, \Delta V + \frac{\Delta m v_2^2}{2} + \Delta m g h_2 = P_1 \cdot \Delta V + \frac{\Delta m v_1^2}{2} + \Delta m g h_1\,, \quad \textit{or after dividing by } \Delta V:$$

$$P_2 + \frac{\rho v_2^2}{2} + \rho g h_2 = P_1 + \frac{\rho v_1^2}{2} + \rho g h_1$$

Finely:

Bernoulli's equation states that for an incompressible, frictionless fluid

$$P + \frac{\rho v^2}{2} + \rho g h = const$$

Where:

- **P is an absolute inner static pressure**
- **ρ is the fluid density**
- ***v* is the velocity of the fluid**
- **h is the height above some reference point**,
- **g is the acceleration due to gravity**

$\frac{\rho v^2}{2}$ **- hydrodynamic pressure**,

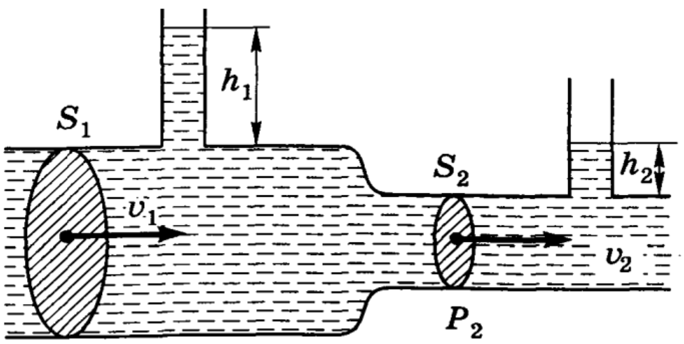
$\rho g h$ **- hydrostatic pressure.**

The Bernoulli Equation can be considered to be a statement of the conservation of energy principle appropriate for flowing fluids. Conservation of energy applied to fluid flow produces Bernoulli's equation.

The net work done by the fluid's pressure results in changes in the fluid's kinetic and potential energy per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth:

$$\rho v_1^2/2 + P_1 = \rho v_2^2/2 + P_2$$

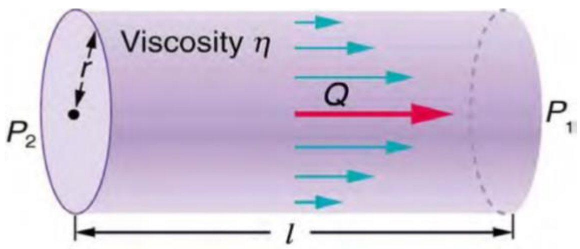


The pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if *v*2 is greater than *v*1 in the equation, then *P*2 must be less than *P*1 for the equality to hold.

3. Flow of viscous fluid in tube. Poiseuille's law.

In steady flow the fluid in a tube is not accelerating, and an equation relating the pressure gradients to the viscous shear stresses can be derived by consideration of the equilibrium of an annulus of the fluid.

For the Newtonian fluid the velocity profile for laminar flow in a tube is parabolic. The velocity is zero at the wall and reaches a maximum at the centerline.



Poiseuille's law:

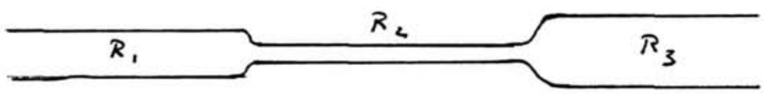
$$Q = (P_2 - P_1)\frac{\pi r^4}{8\eta l} \quad \text{Or} \quad Q = \frac{(P_2 - P_1)}{R_h}, \quad Or \quad (P_2 - P_1) = \frac{8\eta l}{\pi r^4} Q$$

where $R_h = \dfrac{8\eta l}{\pi r^4}$ – hydro /hemo dynamic resistance.

Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity η through a tube of length l and radius r . The direction of flow is from greater to lower pressure. Flow rate Q is directly proportional to the pressure difference $P2 - P1$, and inversely proportional to the length l of the tube and viscosity η of the fluid. Flow rate increases with $r4$, the fourth power of the radius.

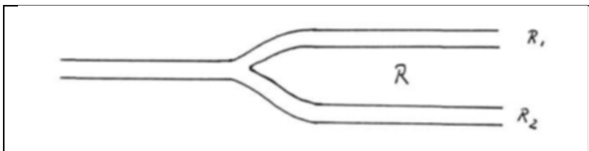
4. Hydrodynamic resistance of a system of tubes

4.1 Tubes in series



$$R = R_1 + R_2 + R_3$$ (resistance adds up) – в скобках убрать

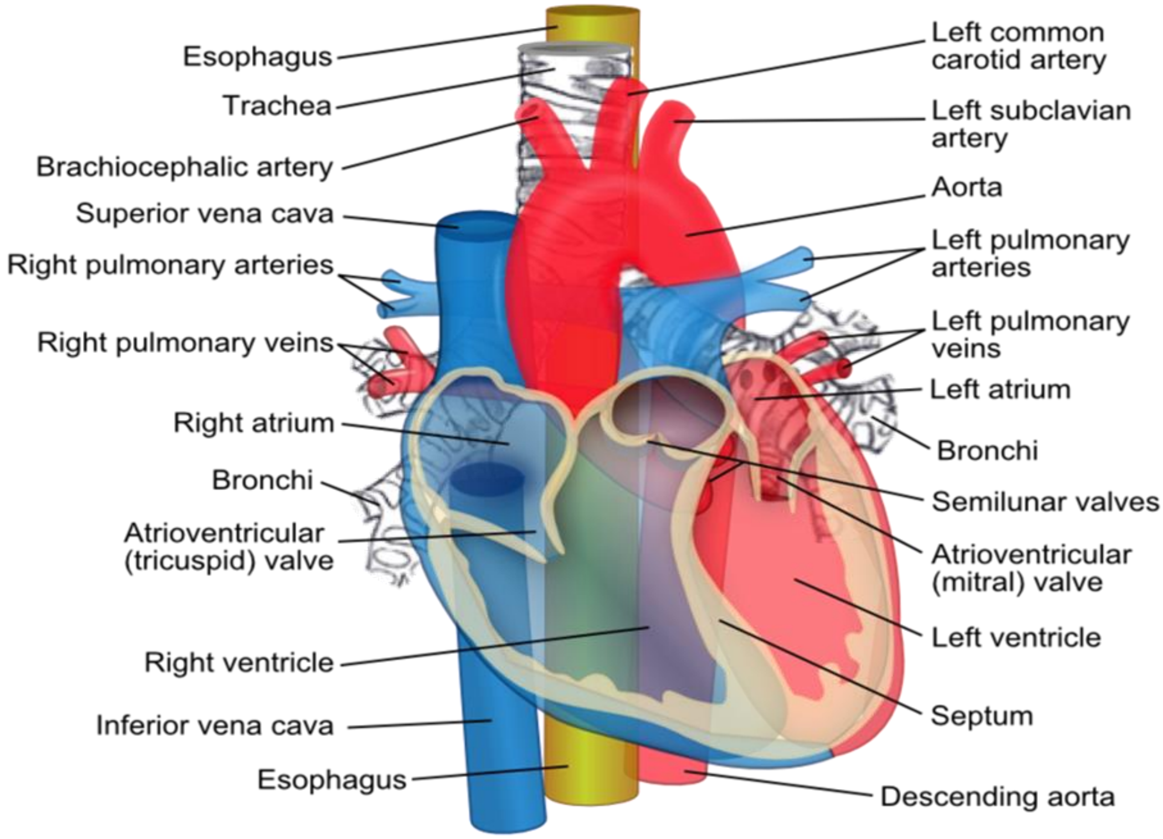
4.2 Tubes parallel

	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Part II. The Basis of Hemodynamics

1. The cardiovascular system: Heart – Arteries – Veins

1. 1 Heart – double pump system; Arteries – are blood vessels which transport blood away from the heart; Veins - are blood vessels which transport blood towards the heart



4 Chambers: Left Atrium – Left Ventricle (Left heart) – Right Atrium – Right Ventricle (Right Heart)

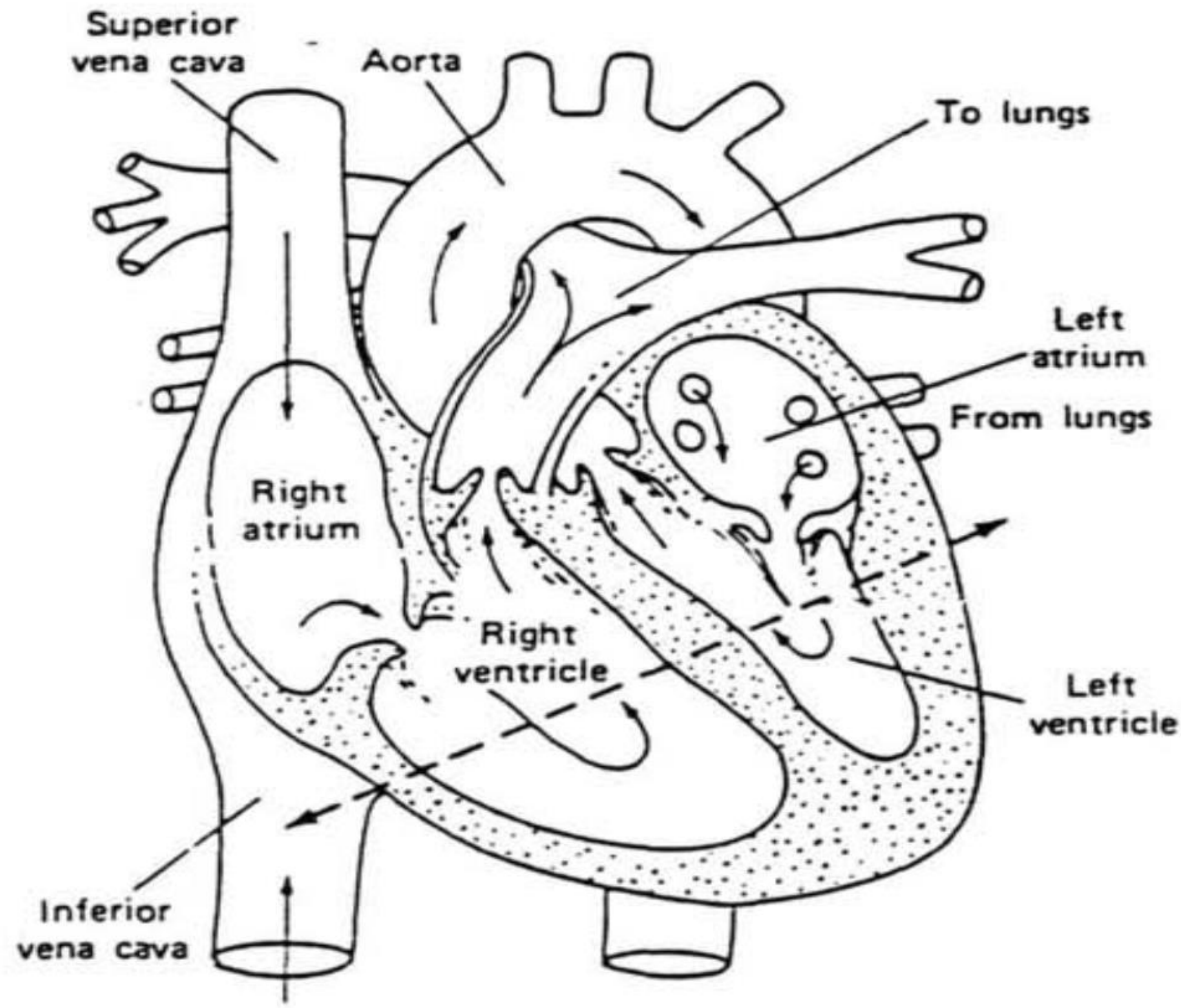
4 Valves – are controlled by the pressure conditions in the ventricles caused by the contraction of the heart muscle – increase in pressure closes inlet valve and opens outlet valve. In Such a way valves determine the flow direction of the blood.

2 atrioventricular valves

- The mitral (bicuspid) valve lies between the left atrium and left ventricle
- The tricuspid valve lies between the right atrium and the right ventricle

2 semilunar valves sit at the exit of each of the ventricles

- The aortic valve
- The pulmonary valve



- Left ventricle chamber is compressed by heart muscle (P=125 mm Hg)

- Blood is pumped through the Aorta into the large Systemic Cycle which closes at right atrium (2 veins – Superior Vena Cava and Inferior Vena Cava)

- Blood is pumped by weak contraction (P=6 mm Hg) to right ventricle chamber

- Right ventricle chamber compresses (P=25 mm Hg) and pumps blood through 4 pulmonary arteries into the small Pulmonary Cycle which closes at left atrium

- The left atrium receives oxygenated blood back from the Pulmonary Cycle via one of the 4 pulmonary veins

- Blood is pumped by weak contraction (P= 8 mm Hg) to left ventricle.

The amount of work done by heart (heart muscle) can be calculated from the force $\boldsymbol{F}$ developed by heart muscle and linear change in the heart chamber size:

$$W_h = \int F \cdot dl$$

or in terms of blood pressure on the surface area of the heart chamber and the change of its volume:

$$W_H = \int \frac{F}{S} \cdot Sdl = \int_{V_{ES}}^{V_{ED}} P(V) \cdot dV = P_{mean} \cdot \Delta V$$

where:

$P(V)$ – the blood pressure in a ventricle with respect to its volume;

V_{ES} - **End-systolic volume** (ESV) - is the volume of blood in a ventricle at the end of contraction, or systole, and the beginning of filling, or diastole (the lowest volume of blood in the ventricle at any point in the cardiac cycle);

V_{ED} - **End-diastolic volume (EDV) -** is the volume of blood in a ventricle at the end of diastole or filling in, or the amount of blood in a ventricle just before systole;

P_{mean} – the mean blood pressure in the ventricles;

ΔV - the pumped blood volume per beat.

The volume of the pumped blood during each compression is called *Stroke Volume (SV)*:

$$SV = \Delta V = V_{ED} - V_{ES} = EDV - ESV$$

The Frank–Starling law: The stroke volume of the heart increases in response to an increase in the volume of blood filling the heart (the end diastolic volume) when all other factors remain constant.

An **ejection fraction** (EF) is the fraction of blood ejected from a ventricle of the heart with each heartbeat

$$\text{EF} = \frac{\Delta V}{V_d} = \frac{SV}{EDV} = (66 – 67)\ \%$$

Cardiac output Q **(CO) –** the volume of blood being pumped by a ventricle, per unit of time:

$$Q = \Delta V \cdot f = SV \cdot H_r$$

The normal range for CO is 4.0 – 8.0 (L / min)

The work done by heart: $W_H = P_{mv} \cdot \Delta V = P_{ma} \cdot \Delta V + \frac{\Delta m v^2}{2}$

$$P_{mv} - P_{ma} = \frac{\rho v^2}{2}$$

The work done by left and right heart is not equal. It derives from practice that:

$$W_{RH} \approx 0.2\ W_{LH} \text{ then } W_H = W_{LH} + W_{RH} = 1.2 W_{LH}$$

2. Blood Volume, Blood Pressure and Blood Flow Velocity Distribution

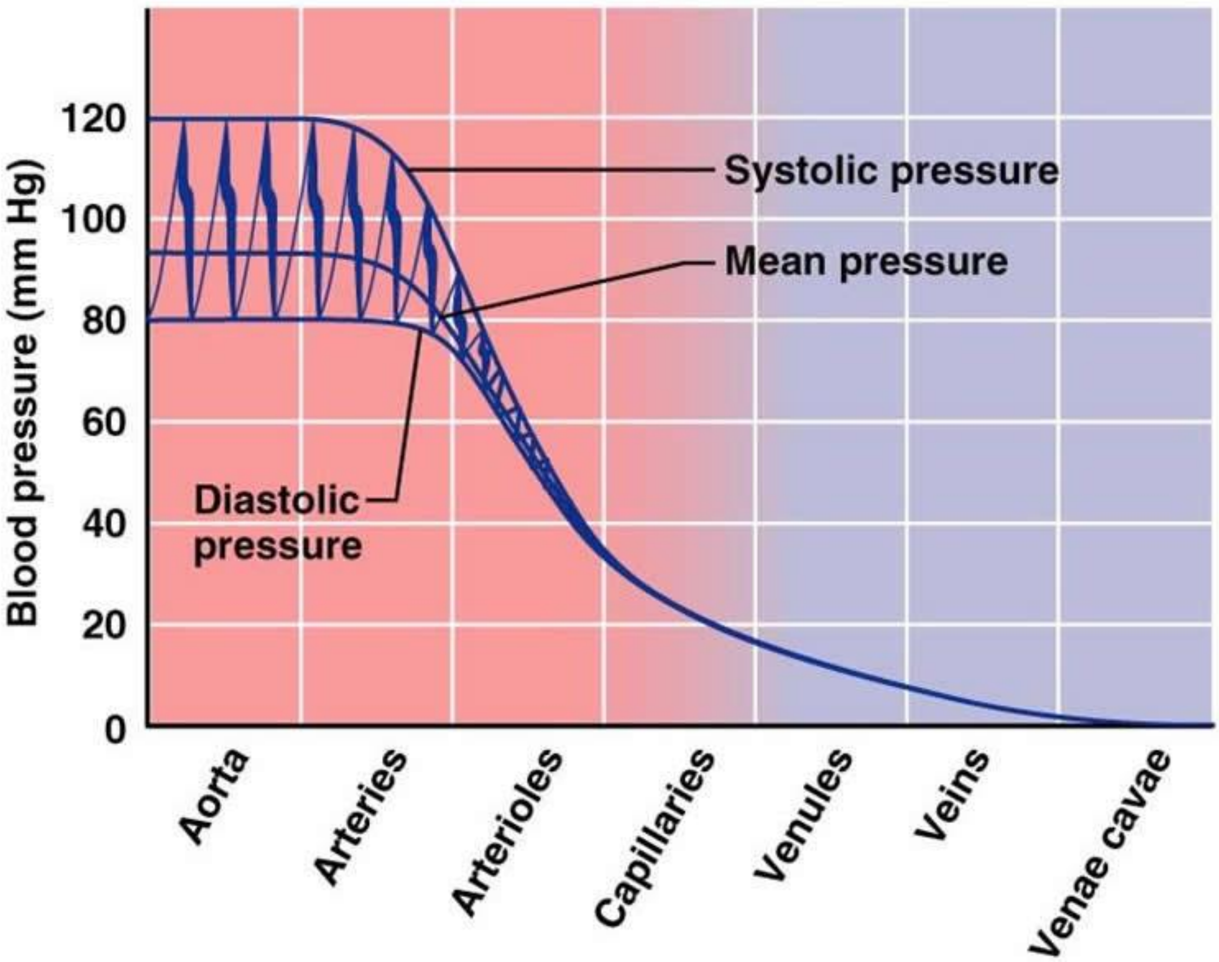
2.1 Blood Volume Distribution

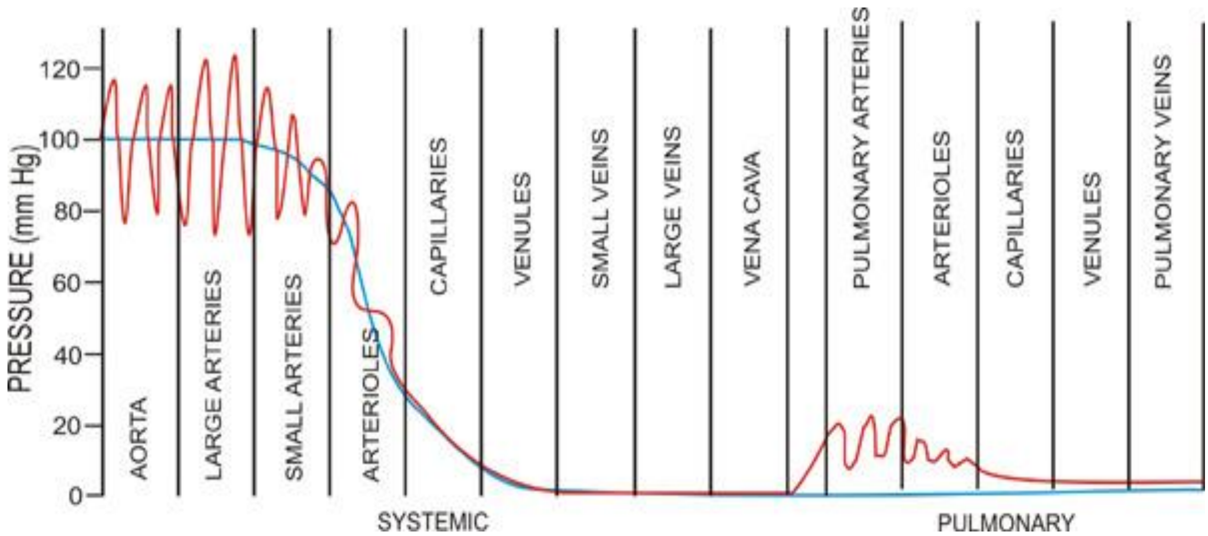
Heart	Systemic Cycle	Pulmonary Cycle
7.2 %	84 %	8.8 %
arteries	6.0	
arterioles	8.0	2.6
capillaries	6.0	2.2
small veins	46.0	
large veins	18.0	4.0

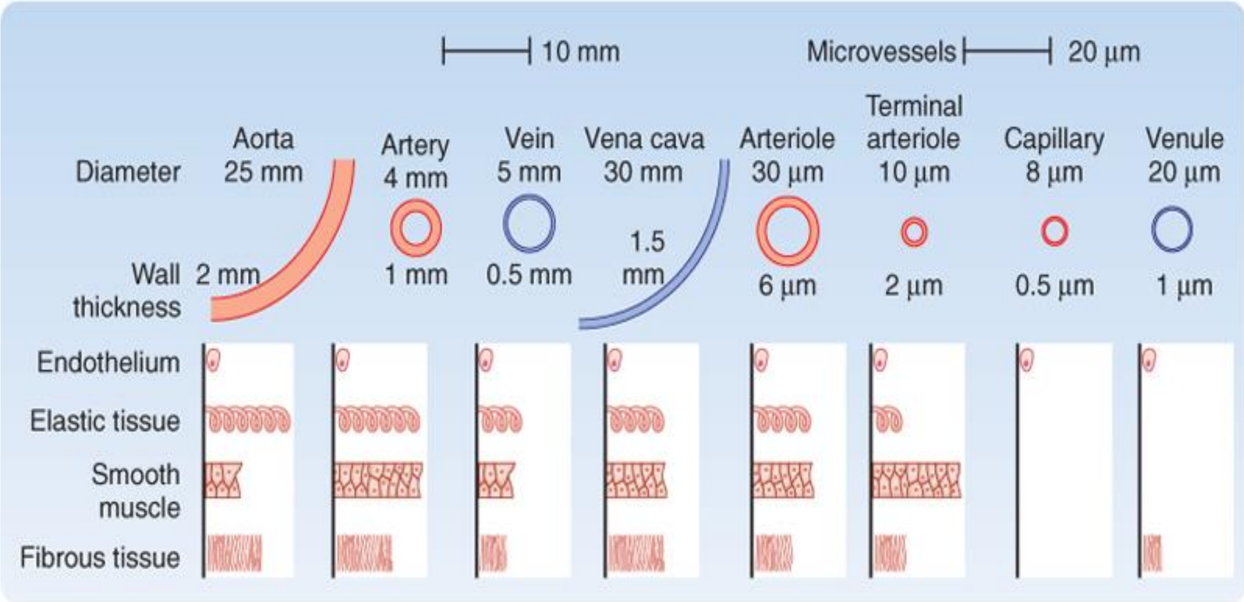
The total blood mass is ≈ 7% of the total body mass. The blood density ρ = 1.05 $\frac{g}{sm^3}$

2.2 Blood Pressure and Blood Flow Velocity Distribution

Region	Blood Pressure (mmHg) Ps / Pd and Pav	Average Blood Velocity
Left Ventricle	120 / 10	
Aorta	120 / 70	0.3 m/s (30 cm/s)
Artery	Pav 95 - 90	
Arterioles	70 - 35	
Capillaries	30 - 15	$(1 - 0.3) \cdot 10^{-3} \frac{m}{s}$ (0.3 mm/s)
Small vein	15 - 10	
Vena Cava	10 - 4	0.15 – 0.4 (m/s) (15 cm/s)
Right Atrium	4 - 5	
Right Ventricle	25 / 5	
Pulmonary Artery		63 cm/sec


Blood pressure (mm Hg)
120
100
80
60
40
20
0
Systolic pressure
Mean pressure
Diastolic pressure
Aorta
Arteries
Arterioles
Capillaries
Venules
Veins
Venae cavae


PRESSURE (mm Hg)
120
100
80
60
40
20
0
AORTA
LARGE ARTERIES
SMALL ARTERIES
ARTERIOLES
CAPILLARIES
VENULES
SMALL VEINS
LARGE VEINS
VENA CAVA
PULMONARY ARTERIES
ARTERIOLES
CAPILLARIES
VENULES
PULMONARY VEINS
SYSTEMIC
PULMONARY



Koeppen & Stanton: Berne and Levy Physiology, 6th Edition.

The model of a vascular bad:

1. All respective regions of a vascular bed (arteries, arterioles, capillaries, venules, veins) are connected in series. Then the total blood resistance of the vascular bad is equal to the sum of resistances of each region (resistance adds up).

2. All vessels in a respective region of the vascular bed are connected in parallel: all arteries are parallel, all arterioles are parallel, all capillaries are parallel, all venules are parallel, all veins are parallel.

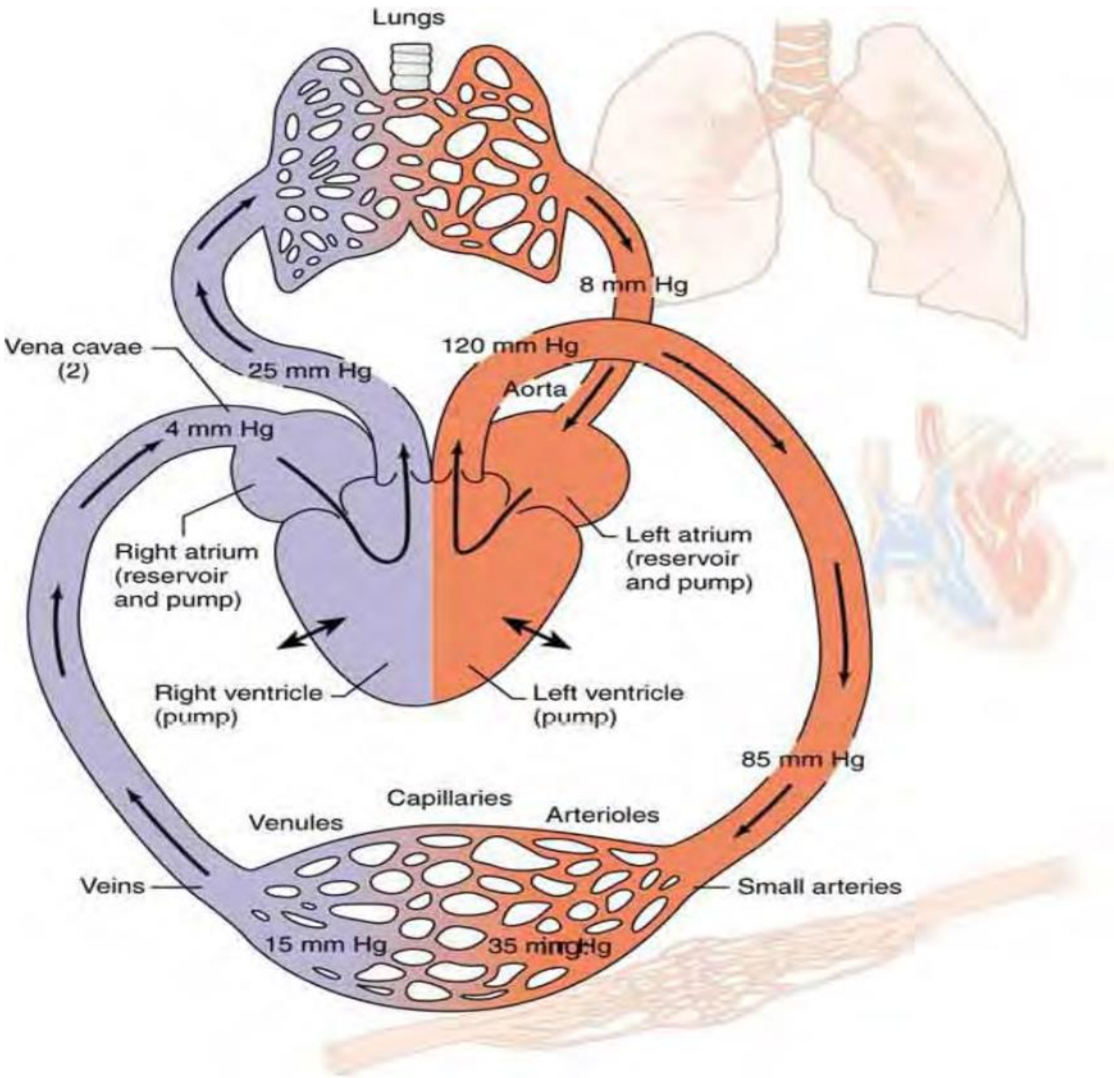
3. All respective vessels have the same length and diameter and respectively the same hemodynamic (blood) resistance.

Under above made assumptions the total vascular resistance (VR) (total peripheral resistance (TPR)) is equal to:

$$R = \frac{R_{artery}}{n_{arteries}} + \frac{R_{arteriole}}{n_{arterioles}} + \frac{R_{capillary}}{n_{capillaries}} + \frac{R_{vein}}{n_{veins}}$$

Schematic of the circulatory system

Blood Pressure Distribution (Gauge pressure in mm Hg)



Schematic of the circulatory system. Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

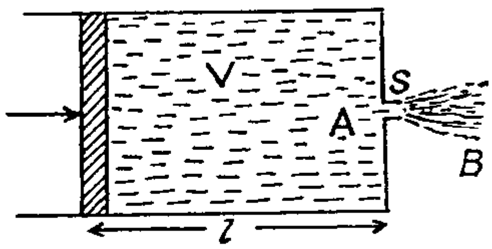
II. Test Problems

1. What work should be done in order to squeeze all water from a horizontally located cylinder (Fig. 1.85) during the time t by means of a constant force acting on the piston? The volume of water in the cylinder is equal to V, the cross-sectional area of the orifice to s, with s being considerably less than the piston area. The friction and viscosity are negligibly small.

Let the force acting on the piston be F and the length of the cylinder be l.

Then, work done $= Fl$ (1)

Applying Bernoulli's theorem for points A and B, $p = \frac{1}{2}\rho v^2$ where ρ is the density and v is the velocity at point B. Now, force on the piston,



$$F = pA = \frac{1}{2}\rho v^2 A \qquad (2)$$

where A is the cross section area of piston.

Also, discharge through the orifice during time interval $t = Svt$ and this is equal to the volume of the cylinder, i.e.,

$$V = Svt \quad \text{or} \quad v = \frac{V}{St} \qquad (3)$$

From Eq. (1), (2) and (3) work done

$$= \frac{1}{2}\rho v^2 A l = \frac{1}{2}\rho A \frac{V^2}{(St)^2} l = \frac{1}{2}\rho V^3/S^3 t^2 \quad (\text{as } Al = V)$$

2. Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

3. A major artery with a cross-sectional area of 1.00 cm2 branches into 18 smaller arteries, each with an average cross-sectional area of 0.400 cm2 . By what factor is the average velocity of the blood reduced when it passes into these branches?

4. (a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is 10.0 cm2 , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is 10.0 μm ?

5. The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about 8 μm . Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

6. The flow rate of blood through a 2.00×10−6-m -radius capillary is 3.80×109 cm3 /s .

(a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.)

(b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of 90.0 cm3 /s ? (The large number obtained is an overestimate, but it is still reasonable.)

7. The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the flow is laminar. The average speed of the blood is 15mm/s.

(a) What is the net force on the blood in this section of artery?

(b) What is the power expended maintaining the flow?

8. A small artery has a length of $1.1 \times 10{-}3$ m and a radius of $2.5 \times 10{-}5$ m . If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is 37° C .)

9. The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.

10. Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?

11. (a) Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase?
(b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

12. The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm , calculate the number of capillaries in the blood circulatory system.

13. How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min? $V = 2.0\ \ 10^5\ m3$ This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

14. When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?

15. During a marathon race, a runner's blood flow increases to 10.0 times her resting rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

Reading:

- Medical and Biological Physics. Textbook for students of higher medical education institutions / V.I. Fediv, O.I. Olar, V.V. Kulchynskyi, H. Y. Rudko. – Chernivtsi: BSMU Publishing House, 2017. - 342 p. – English.

- Medical Physics. https://www3.nd.edu/~nsl/Lectures/mphysics/

Watching:

https://www.youtube.com/watch?v=diG519dFVNs