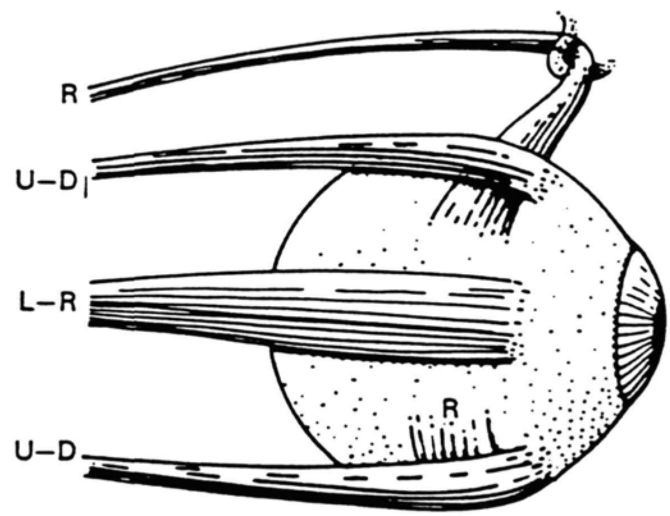
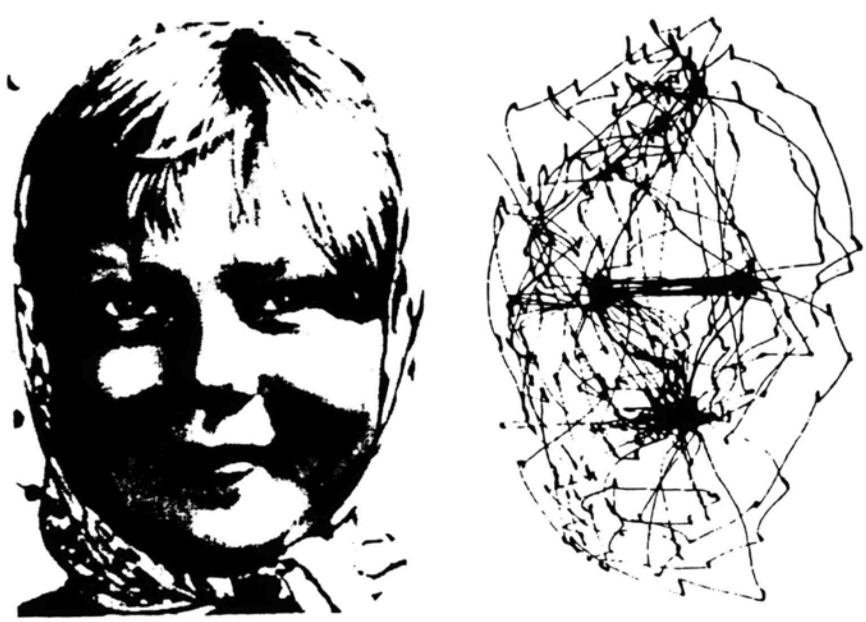
5. THE OPTICAL SYSTEM OF THE BODY

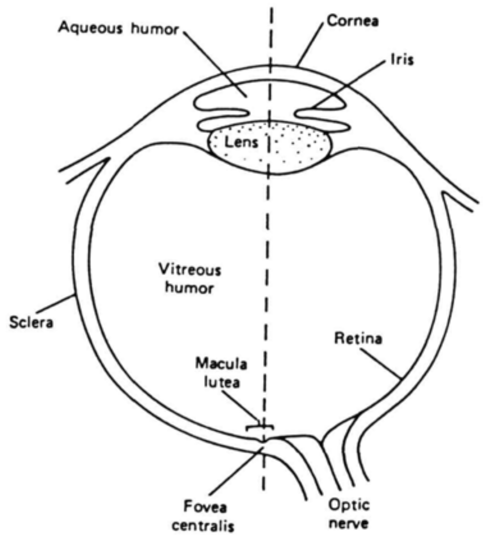
5.1. THE PHYSICS OF THE EYE





General structure of the eye

The human eye is a adjustable lens system consisting out of two focussing elements and a light receptor system.



- *cornea* transparent thick layer in front of the eye-ball
- *iris-pupil* aperture system
- *lens* crystalline lens with variable radii
- *retina* light receptor system
- *fovea centralis* sensitive part of receptor system
- *optic nerve* information tarnsport system

The inside of the eye is filled with liquid material

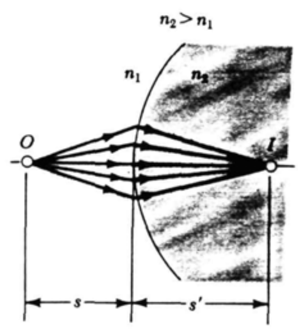
- first eye chamber: aqueous humor, n_{ah}=1.33
- second eye chamber: vitreous humor, n_{vh}=1.33

Refractive focussing of the eye system

The cornea is the main focussing element with a front radius R $\equiv$ to first lens in a double lens system.

Material has fairly high index of refraction, n_c=1.37 which favors refractive focussing from the low index of refraction of air, n_{out}=1.

Approximate constant index of refraction n_{in} inside of eye $\Rightarrow$ focussing inside single thick lens.



Relation between object distance s and image distance s':

$$\frac{n_{out}}{s} + \frac{n_{in}}{s'} = \frac{n_{in} - n_{out}}{R}$$

The focal lengths of the cornea are:

$$f_{front} = \frac{n_{in} \cdot R}{n_{in} - n_{out}}$$

$$f_{back} = \frac{n_{out} \cdot R}{n_{in} - n_{out}}$$

Radius of the cornea R≈7.5 mm.

f_{front} ≈27-30 mm
f_{back} ≈20-22 mm

This is close to the the typical diameter of the eye ball ≈ 25 mm.
This represents the image distance s'

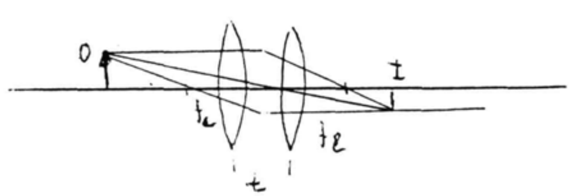
Focal length under water: n_{out}=1.33

f_{front} ≈256 mm
f_{back} ≈250 mm

Focussing capabilities are reduced, main focussing is done by second lens.

The dioptric strength of the eye D=1/f [m]; as larger focal length as smaller the dioptric strength.
For f=100 cm ⇒ D=1 dioptrie (converging lens)
for f=100 cm ⇒ D=-1 dioptrie (diverging lens)

for f_{cornea}=27 mm ⇒ D=37 dioptrie
The main focussing is performed by the cornea, the lens doesthe fine-tuning.
to obtain sharp focussing at the retina (short image distance)



Eye is a double lens system, therefore the total focal length of the eye

f_e and the dioptric strength are:

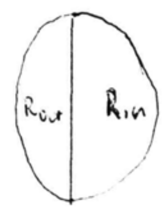
$$\frac{1}{f_c} + \frac{1}{f_l} = \frac{1}{f_e}$$

$$\mathrm{D}_e = \mathrm{D}_c + \mathrm{D}_l$$

If lenses are separated by distance t:

$$D_e = D_c + D_l - \frac{t}{f_c \cdot f_l} = D_c + Dl - t \cdot D_c \cdot D_l$$

The lens has variable radii R_{out} and R_{in}



$$D_l = \frac{1}{f_l} = \left(\frac{n_l}{n_{ah}} - 1\right) \cdot \left(\frac{1}{R_{out}} - \frac{1}{R_{in}}\right)$$

with the index of refraction $n_l \approx 1.4$, and the radii (adjustable)
R_{out} = 5.3 mm (near accomodation) to 10.0 mm (far accomodation)
R_{in} = -5.3 mm (near accomodation) to -6.0 mm (far accomodation)

$\Rightarrow$ D_l=14 dioptrie (far accomodation), f_l=71.3 mm $\Rightarrow$
D_l=20 dioptrie (near accomodation), f_l=50.4 mm

With a distance of $t \approx 5$ mm between cornea and lens, this results in a total dioptric strength of:
D_e=48 to 53 dioptrie (far to near accomodation) which results in a total focal length of 20 mm - 21 mm

Near focussing is limited by the curvatures of the cornea and the adjustable radii of the lens.
With the simplifying assumption for a single thin lens:

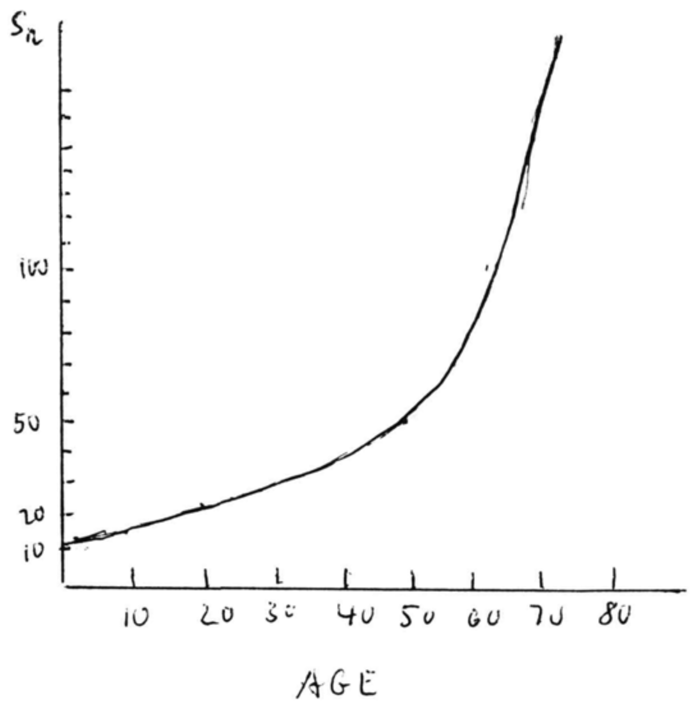
$$\frac{1}{s} + \frac{1}{s'} = D_e$$

$$s = \frac{1}{(D_e - \frac{1}{s'})}$$

Therefore minimum object distance where sharp focussing is possible is determined by the largest achievable dioptric strength of the eye.

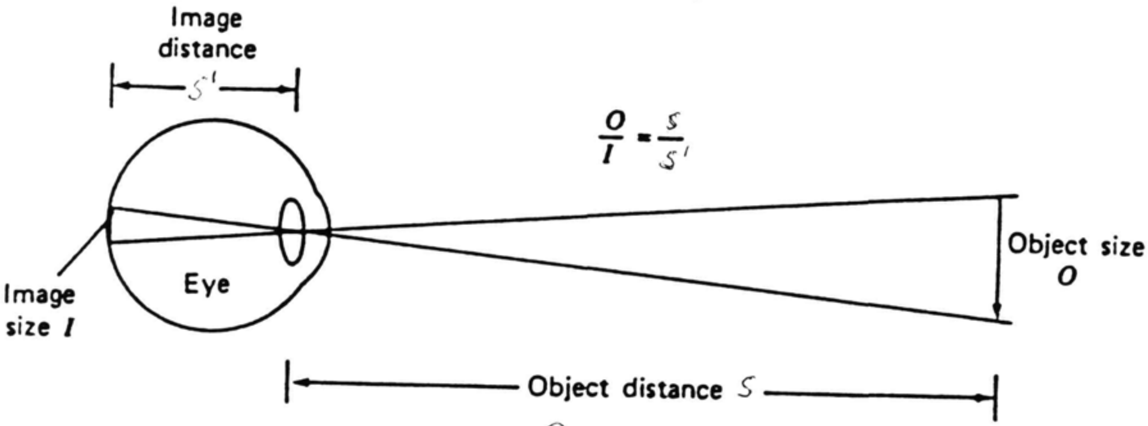
for $D_e \approx 53$, $s' \approx 2.5$ cm $\Rightarrow$ $s_{near} \approx 8$ cm

Only in very young age achievable, for average person the near point is around 20-25 cm. near point distance increase with age.



Geometrical optics of the eye

Object - image relation for the eye can be approximated by a simple quasi-one lens system with the image distance $s' \approx$24 cm.



$$\frac{O}{I} = \frac{s}{s'}$$

$$I = O \cdot \frac{s'}{s} = O \cdot \frac{2.4\ cm}{s\ [in\ cm]}$$

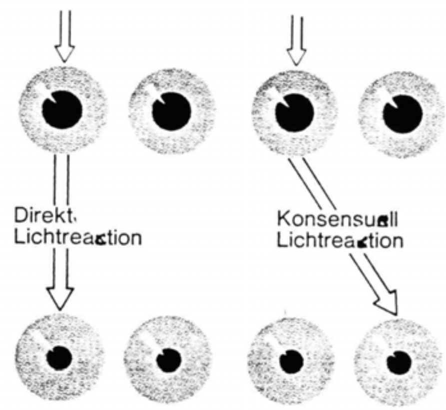
Example A 2 m person at a distance of 10 m has an image height of $I \approx$0.5 cm; at 1 m distance: $I \approx$5.0 cm

The latter image height is larger than diameter of the eye; only part of the person can be viewed at a time.

The pupil operates as aperture of the eye,
opening adjustemnt depends on the intensity of the light.

Adjustment regulates that the absorbed light intensity I_{light} remains below critical value at the retina, therefore:

$$I_{light} \cdot A_{pupil} \leq C$$



The aperture area of the pupil can vary between 1.8 mm^2 and 51 mm^2

The response time of the iris with respect to changes in light intensity is ≈0.3 - 0.8 s

Structure of the receptor system

The backside of the eyeball is called retina (area $\approx$ 50 - 60 cm^2)
The retina is covered with light sensitive photo receptor systems (which convert by photo-induced chemical reactions the light into electrical energy)

receptors are;
cones, responsible for daylight absorption, sensitive to fine detail and color - cones have direct nerve connection to brain

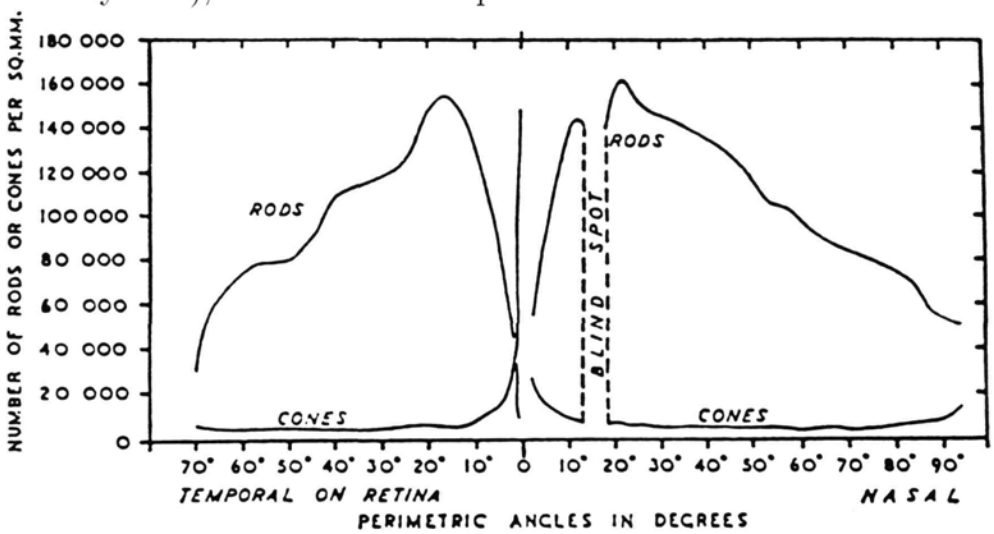
rods, responsible for night and peripheral vision - nerve connection per multitude of rods, therefore sensitive at low light intensity

cones and rods are covered by nerve tissue except in small center area $\approx$0.3 mm^2 (diameter 3 mm), the fovea centralis (yellow spot). area of high sensitivity

fovea centralis determines the size for sharp images and therefore limits the view to only small objects; to see a 2 m man well focussed requires a object distance of :

$$s = \frac{O \cdot 2.5\ cm}{I} = \frac{2\ m \cdot 2.5\ cm}{0.\cancel{0}3\ cm} = 16.6\ m$$

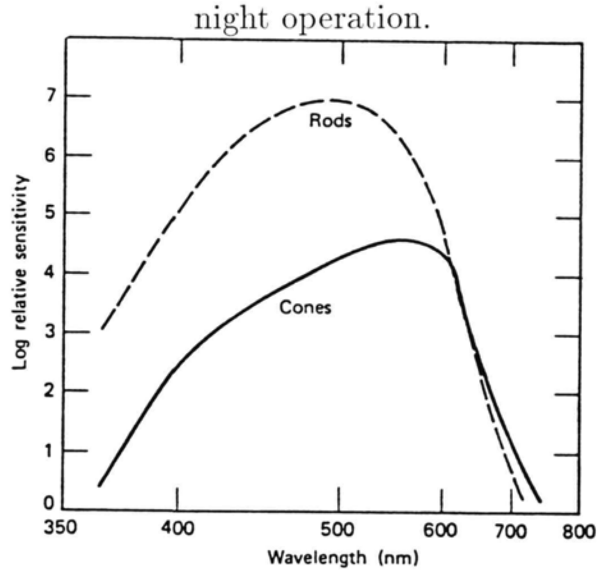
No homogeneous distribution of rods and cones, rods are distributed over entire retina area (except blind spot - exit for optical nerve system); number of cones peak near the fovea centralis.



number of cones: $\approx 65\ 10^6$/eye

number of rods: $\approx 120\ 10^6$/eye

different sensitivity to wavelength, rods are more sensitive - better night operation.

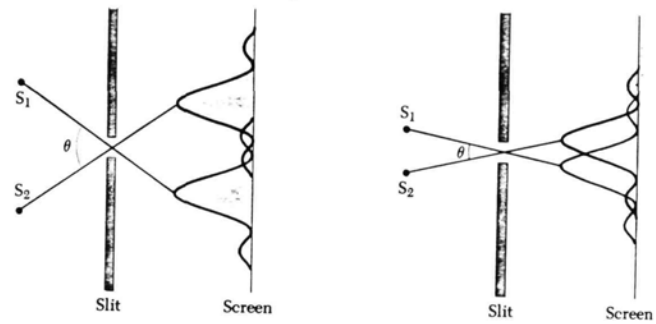


Compared to iris response to change in light intensity, receptor response is relatively slow (chemical process)

Adaptation takes place within 5 to 30 minutes. Therefore slow adjustment to sudden darkness.

Diffraction effects of the eye

Due to the wave nature of light diffraction effects occur on small apertures.
The resulting diffraction pattern limits the resolution of the optical system.



According to Rayleighs Criterion two points can be resolved with an aperture system of diameter a if:

$$\alpha = \frac{1.22 \cdot \lambda}{a} \approx \frac{O}{s}$$

for $\lambda \approx 550$ nm and $a \approx 8$ mm (maximum opening)
$\Rightarrow \qquad \alpha \approx 8.4 \cdot 10^{-5}$ radians

At s=1 m distance objects of the size of O=84 μm can be resolved, in 10 m distance objects of $\approx$1 mm can be resolved.
For a "normal" opening of the pupil a=3 mm, at 10 m distance a 2.2 mm object can be resolved.

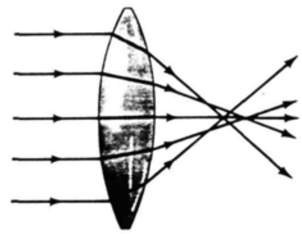
The image spot at the retina (at distance d$\approx$2 cm from pupil) (for a=3 mm) is:

$$I = d \cdot 2\alpha \approx 4.5 \cdot 10^{-4}\ cm = 4.5\ \mu m$$

the cones are spaced with 2μm distance comparable to optical resolving power.

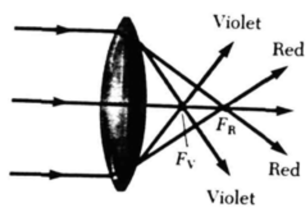
Optical aberrations of lenses

spherical aberrations are due to differences in focal length for light rays at the outer edges of the lens and light rays at the center of the lens.



In human eye this effect is corrected by small pupil size which cuts out light from the outer edges of the cornea.

chromatical aberrations are due to wavelength dependence of the index of refraction; light with different wavelength has different focal length.



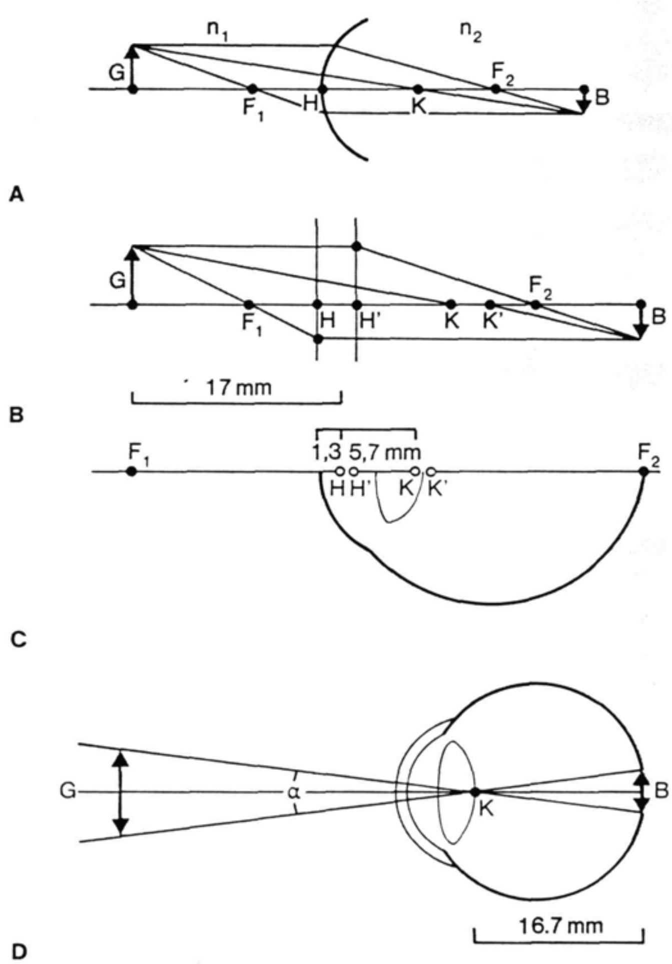
As a consequence at identical distance blue objects appear to be further away than red objects.
(Medieval church windows often show blue background and foreground figures in red to achieve optical depth - three dimensional effect → Notre Dame church)

Eye defects

Normal eye ≡ emmentropic eye

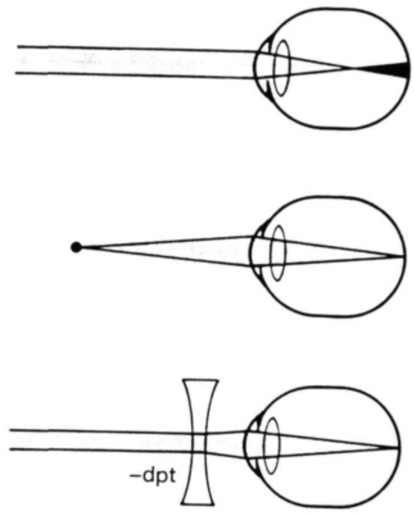
- relaxed (focus on far distance object): D≈50
- focused at near point : D≈54
- near point ≈25 cm (age dependent)
- far point (greatest distance at which good focus can be obtained): infinity

Any defects from normal sight are called ametropia



Myopia - near sightedness

Axial length of eye is to long - deformation along optical axis
$\Rightarrow$ Focus point is in front of retina
Far point is closer than normal



Requires correction with a diverging lens

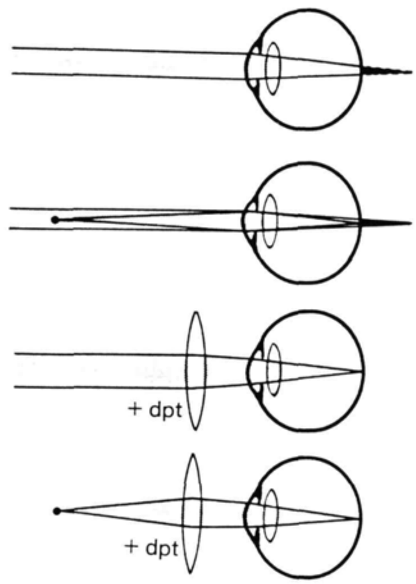
Example Myopic eye has a far point of s_f=1 m (object at larger distances are not focused), what strength for the corrective lens is required? Take for the image distance $s' \approx$2 cm.

$$D = \frac{1}{f} = \frac{1}{s_f} + \frac{1}{s'} = 51$$

compared to normal eye D=50; correction with dispersive lens of D=-1 required.

Hyperopia - far-sightedness

Axial length is too short, $\Rightarrow$ focus point is behind retina
Near point is further away than normal.



Requires correction with converging lens

Example What strength of lens is required if the near point of a far sighted person is at s_n=4 m? The dioptric strength of the normal eye focused at the near point is D=54.

$$D = \frac{1}{f} = \frac{1}{s_n} + \frac{1}{s'} = 50.3$$

A corrective lens of D=54-50.3=+3.7 is required.