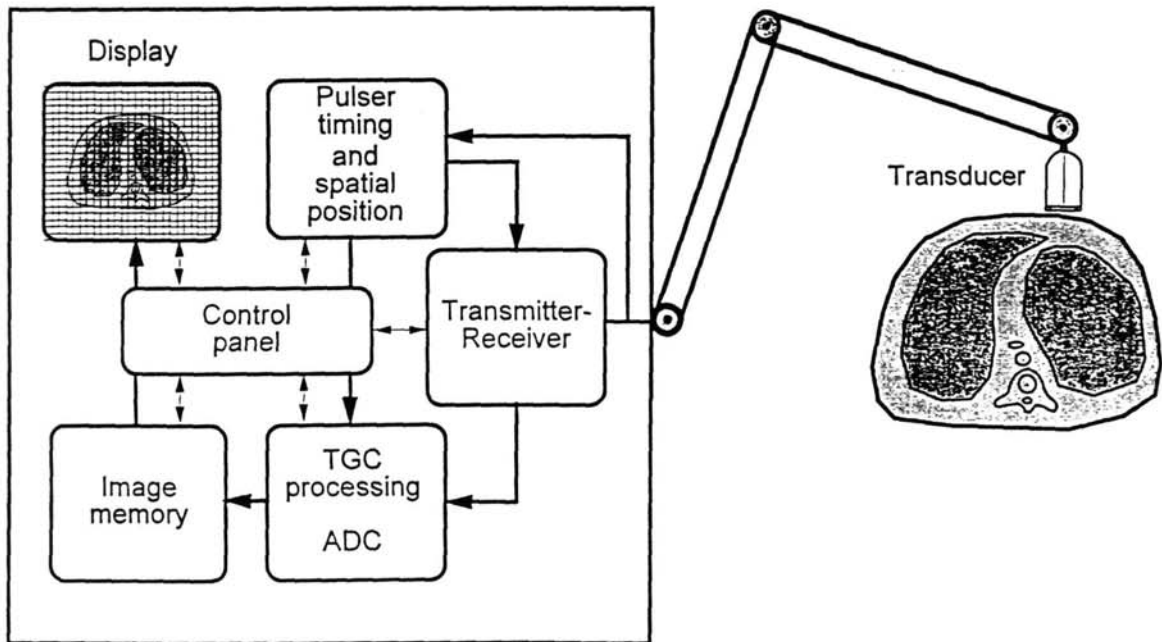


4. THE ACOUSTICS OF THE BODY

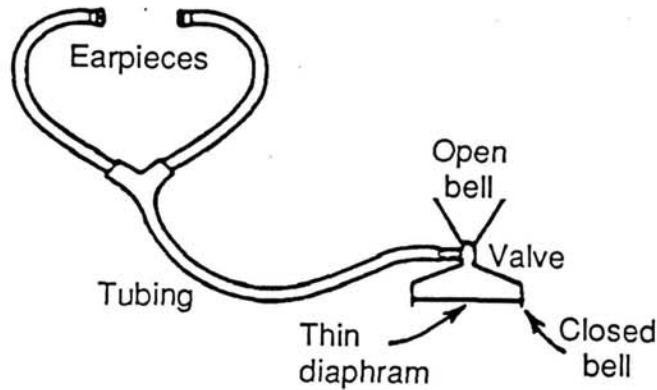
4.4. DIAGNOSTICS WITH SOUND AND ULTRASOUND



THE STETHOSCOPE

The stethoscope is the standard instrument to amplify and analyze characteristic body sounds like heart beat and blood flow (see section 3). (For a history of the stethoscope see Cameron, Skofronick and Grant, 'Physics of the Body'.)

The use of the stethoscope is based on the principle of sound transmission through a tube with both ends closed.



The bell is attached to the body to pick up sounds. The open bell relies on the skin as vibrating diaphragm. The closed bell has its own diaphragm.

The bell serves as impedance matcher between body matter and the air in the tube. This requires that the frequency of the sounds must resonate in the bell membrane. The natural resonant frequency f_{res} of the bell depends on bell diameter d and tension T of the diaphragm with the area density σ [kg/m^2],

$$f_{res} \propto \frac{1}{2 \cdot d} \cdot \sqrt{\frac{T}{\sigma}}$$

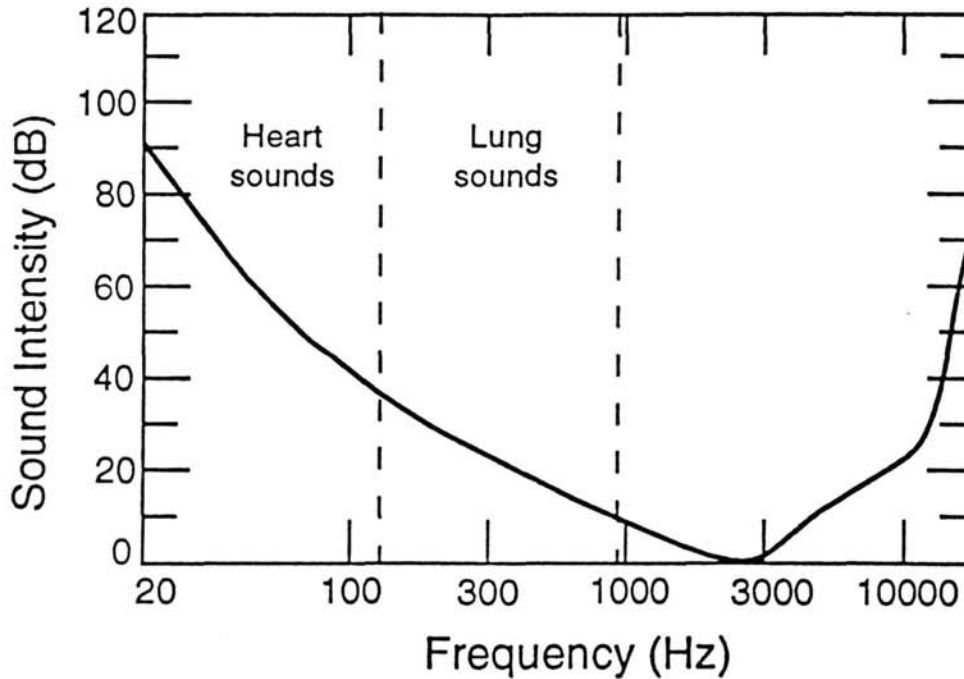
$$f_{res} = \frac{1}{2 \cdot d} \cdot \sqrt{\frac{T}{\sigma} \cdot \beta} = \frac{1}{2 \cdot d} \cdot \sqrt{\frac{P}{\sigma} \cdot \pi \cdot (d/2)^2 \beta} = \frac{1}{4} \cdot \sqrt{\frac{P}{\sigma} \cdot \pi \beta}$$

with β as vibrational parameter $\beta \approx 0.76$.

To selectively pick up certain frequency ranges (low frequency heart murmur, high frequency lung sounds) the appropriate bell size and diaphragm tension must be chosen. For the open bell the tension is modified by changing the pressure of the bell against the skin. With increasing tension the sensitivity for high frequency sounds improves. To select a low frequency range a large diameter bell must be used.

For open bells the frequency range can be selected by the right choice of pressure against the skin

$$P = 16 \cdot f_{res}^2 \frac{\sigma}{\pi \cdot \beta}$$



Typical sounds associated with the heart beat due to blood turbulences are in the 20 to 200 Hz range, sounds associated with the operation of the lung (air turbulences during inhaling and exhaling) are in the frequency range between 200 - 2000 Hz.

EXAMPLE

Calculate the tension necessary for an open bell stethoscope with a bell diameter of 2 cm to pick up a 15 Hz heart murmur. What is the pressure on the skin? Adopt a areal density of $\sigma \approx 0.5 \text{ [kg/m}^2\text{]}$

$$f_{res} = \frac{1}{2 \cdot d} \cdot \sqrt{\frac{T}{\sigma} \cdot \beta}$$

$$T = \frac{4 \cdot d^2 f_{res}^2 \sigma}{\beta} = \frac{4 \cdot 4 \cdot 10^{-4} [\text{m}^2] \cdot 15^2 [1/\text{s}^2] \cdot 0.5 [\text{kg/m}^2]}{0.76} \approx 0.24 [\text{N}]$$

$$P = \frac{T}{\pi \cdot (d/2)^2} = \frac{0.24 [\text{N}]}{3.14 \cdot 1 \cdot 10^{-4} [\text{m}^2]} = 764 [\text{Pa}]$$

What pressure is necessary to hear with the same stethoscope lung sounds of 500 Hz?

$$P = \frac{T}{\pi \cdot (d/2)^2} = \frac{4 \cdot d^2 f_{res}^2 \sigma}{\beta \pi \cdot (d/2)^2}$$

$$P = \frac{4 \cdot 4 \cdot 10^{-4} [\text{m}^2] \cdot 500^2 [1/\text{s}^2] \cdot 0.5 [\text{kg/m}^2]}{0.76 \cdot 3.14 \cdot 1 \cdot 10^{-4} [\text{m}^2]} \approx 840000 [\text{Pa}]$$

This corresponds to a pressure of about 8 bar.

The dimensions of the stethoscope tube must be optimized for good sound transmission without significant intensity losses due to friction of the pressure wave on the tube walls. The volume must be small to maximize the pressure changes along the tube. Impedance matching between diaphragm and pipe is necessary to guarantee optimal transmission of sound.

The acoustic impedance Z of a pipe is the ratio of pressure P to volume velocity U which is the amount of air flowing through a cross sectional area S in $[m^2]$ per $[s]$ due to the sound wave. The acoustic impedance is:

$$Z = \frac{P}{U} = \frac{\rho \cdot v}{S} \approx \frac{400}{S}$$

ρ is the density of the transmitting medium, v is speed of sound. Impedance increase with decreasing pipe radius and changes only above 100 Hz with pipe length. Matching the impedances of the skin, the eardrum and the pipe requires a pipe radius of:

$$r = \sqrt{\frac{S}{\pi}} = \sqrt{\frac{\rho \cdot v}{\pi \cdot Z}} \approx \sqrt{\frac{400}{\pi \cdot Z}} \approx 9 \text{ mm}$$

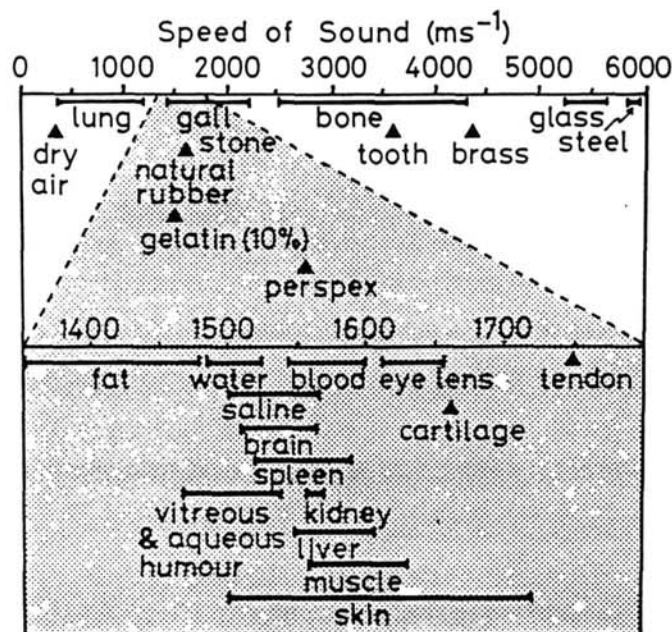
This is too large because at reasonable tube length of $L \approx 30$ cm it would cause intensity losses due to attenuation of the large volume involved. A reasonable diameter is 3 mm.

ULTRASOUND AS DIAGNOSTICAL TOOL

The reflection of acoustical waves at body tissue layers due to impedance mismatch can be used for ultrasound imaging by monitoring the ultrasound echo.

Biological tissues, with the exception of bone and air (lungs) have similar impedances for the transmission of sound, they act like fluids and the speed of sound is given by the density ρ and the elastic modulus K ,

$$v = \sqrt{\frac{K}{\rho}} = 1540 \pm 92 \text{ [m/s]}$$



velocity of sound is frequency independent. This constancy warrants good timing conditions between emission of the ultrasound signal and the detection of the echo signal. This timing allows to probe echos from various depths of body tissue. The spatial resolution Δs depends on the speed and the timing resolution $\Delta t \approx 10 \text{ ns}$ of the electronics.

$$\Delta s = v \cdot \Delta t \approx (1540 \pm 92 \text{ [m/s]}) \cdot 10^{-8} \text{ [s]} \approx 20 \text{ [\mu m]}$$

However spatial resolution is not only limited by timing, but also by wavelength of sound

$$\Delta s \approx \lambda = \frac{v}{f}$$

For low frequencies in the audible range 10 kHz,

$$\lambda \approx 1540 [m/s] / 10^4 [1/s] = 20 [cm]$$

for frequencies in the ultrasound range 10 MHz,

$$\lambda \approx 0.154 [mm]$$

The choice of frequency determines the limits of spatial resolution.

The quality of ultrasound imaging is determined by the interaction of the acoustic wave with the body tissue:

- attenuation depends on absorption and scattering of acoustic waves;
- reflection and transmission depend on the impedances;
- refraction depends on axial direction of acoustic radiation field.

The quality of the image depends on the:

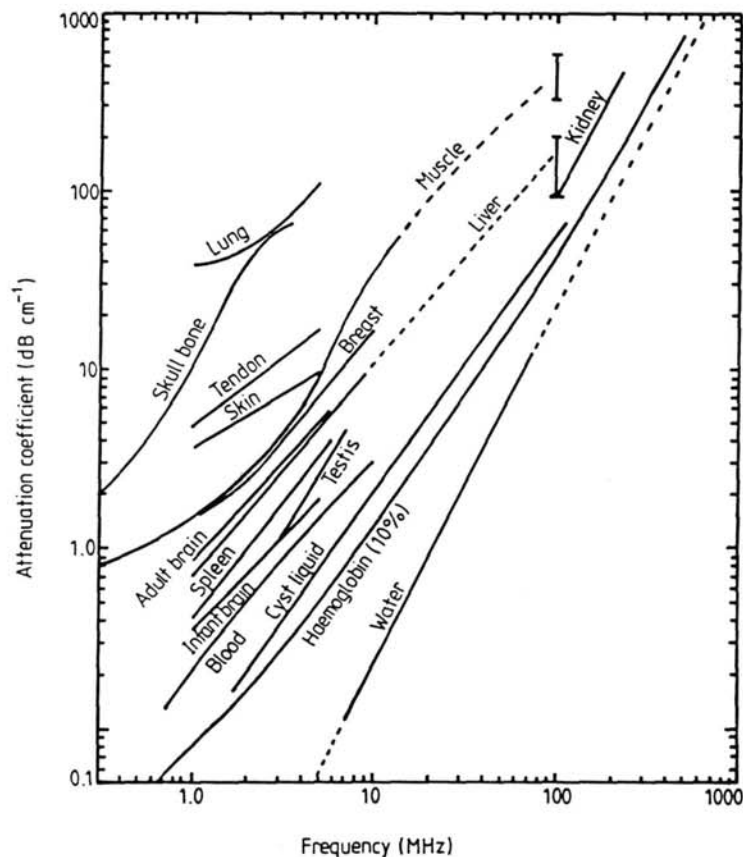
- shape and intensity distribution of the acoustic field of the ultrasound signal
- quality of image generation through data processing

ATTENUATION OF ULTRASOUND

All media attenuate ultrasound due to absorption and scattering effects. The attenuation is exponentially in the direction of the traveling wave and depends on the attenuation coefficient

$$\mu = \mu_{abs} + \mu_{scat}$$

Only total attenuation coefficient can be measured and depends on tissue characteristics (density) and on frequency.



Absorption is due to energy transfer from acoustic wave to vibration of molecules and cells (heating of body tissue).

Scatter of ultrasound takes place on cells or boundaries between different body tissues. Scatter changes the direction of the acoustical waves.

The attenuation for the initial wave intensity I_0 to the intensity I_x at a depth x is:

$$I_x = I_0 \cdot \exp(-\mu \cdot x) \quad \rightarrow \quad \mu = -\frac{1}{x} \cdot \ln\left(\frac{I_x}{I_0}\right)$$

The attenuation coefficient μ [1/cm] is often expressed in terms of decibels ([db/cm]):

$$\mu \text{ [db/cm]} = -\frac{1}{x} \cdot 10 \log_{10}\left(\frac{I_x}{I_0}\right) = -\frac{1}{x} \cdot \ln\left(\frac{I_x}{I_0}\right) \cdot 10 \log_{10} e = 4.343 \cdot \mu \text{ [1/cm]}$$

EXAMPLE

The typical ultrasound signal has an intensity of $2 \cdot 10^5 \text{ W/m}^2$ and a frequency of 10 MHz . This indicates a fat attenuation coefficient of $\mu_{fat} \approx 50 \text{ db/cm}$ after traveling through a 5 cm fat layer the intensity of the acoustical signal is only:

$$I_x = I_0 \cdot \exp(-\mu \cdot x) = 2 \cdot 10^5 \cdot \exp\left(-\frac{50 \cdot 0.05}{4.343}\right) = 1.1 \cdot 10^5 \text{ [W/m}^2\text{]}$$

About 50% of the intensity is absorbed, echo would be mostly absorbed. Probe depth is limited by attenuation, choice of lower frequencies would allow deeper penetration of ultrasound, but would reduce resolution. A frequency of 3 MHz would reduce the attenuation to $\mu_{fat} \approx 2 \text{ [db/cm]}$

$$I_x = 2 \cdot 10^5 \cdot \exp\left(-\frac{2 \cdot 0.05}{4.343}\right) = 1.95 \cdot 10^5 \text{ [W/m}^2\text{]}$$

The attenuation loss is reduced to 2% only, but the resolution decreases to $\Delta s \approx 0.5 \text{ mm}$.

REFLECTION AND REFRACTION

The reflection of a perpendicular acoustic wave is determined by the acoustic impedances of the body tissues.

$$R_p = \frac{I_{ref}}{I_{in}} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$$

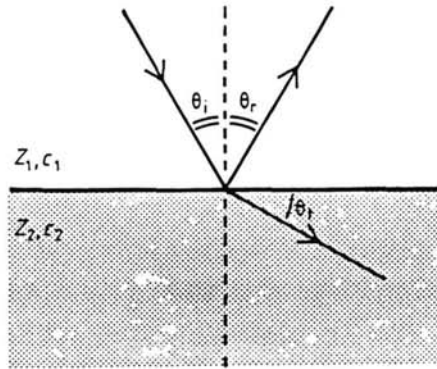
Perpendicular reflection originates the echo signal.

If the wave enters boundary between two body layers at an angle Θ_{in} reflection and refraction occurs. The direction of the reflected Θ_{ref} and refracted signal Θ_{out} is determined by 'Snells law',

$$\frac{\sin \Theta_{in}}{\sin \Theta_{out}} = \frac{v_i}{v_o} = \frac{Z_i \cdot \rho_o}{Z_o \cdot \rho_i}$$

the reflection angle is equal to the incident angle.

$$\Theta_{in} = \Theta_{ref}$$

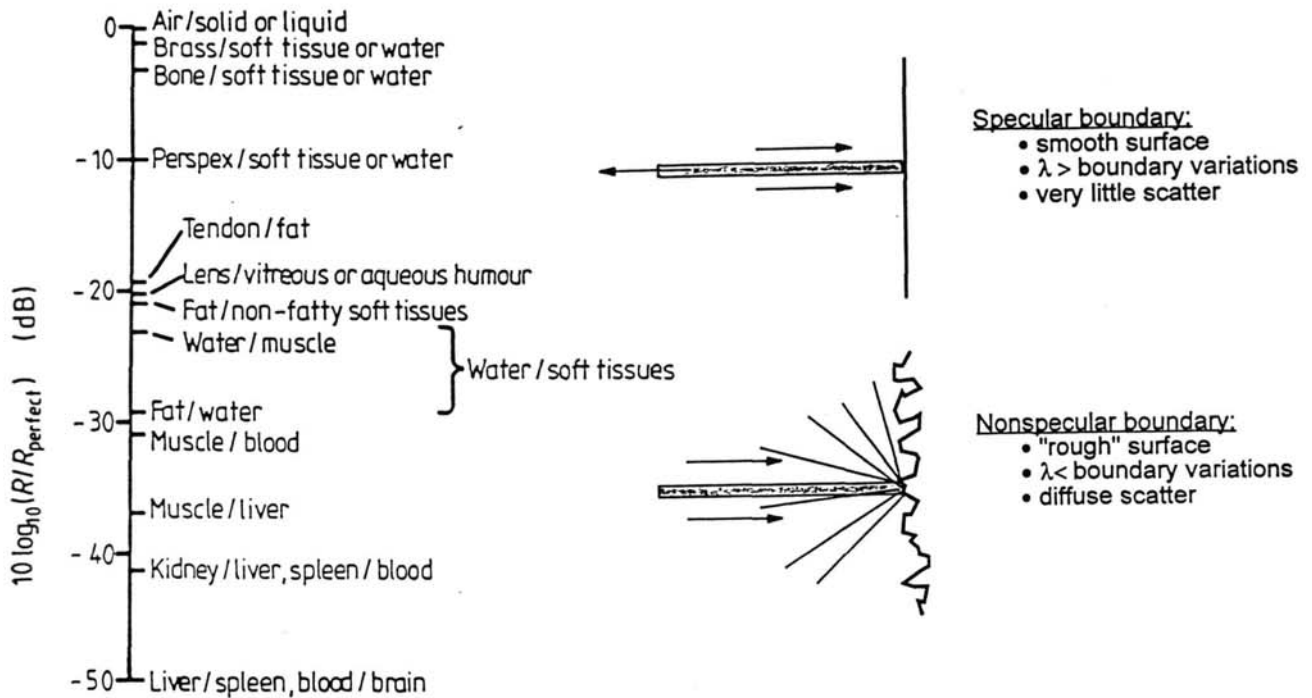


This yields an intensity ratio for nonperpendicular reflection:

$$R_{np} = \frac{I_{ref}}{I_{in}} = \left(\frac{Z_2 \cdot \cos \Theta_o - Z_1 \cdot \cos \Theta_i}{Z_2 \cdot \cos \Theta_o + Z_1 \cdot \cos \Theta_i} \right)^2$$

The intensity loss due to nonperpendicular reflection can be approximated by the coefficient:

$$10 \cdot \log_{10} \frac{R_{np}}{R_p}$$



The calculated reflection coefficients (in db relative to a perpendicular reflector) for a variety of boundaries between biological and nonbiological tissues range between -10 db and -50 db. This corresponds to enormous intensity losses due to insufficient reflection of sound wave.

On the other hand, insufficient reflection means good transmission, deeper layers of the body can be reached. Limitations are layers with large reflection (air-filled cavities) which cannot be probed due to the nearly 100 % reflection of the sound wave at the boundary.

EXAMPLE

What is the approximate intensity loss of a 5 MHz ultrasound wave of traveling to a depth of 4 cm in liver and reflecting on an encapsulated blood clot.

The attenuation coefficient is ≈ 5 [db/cm]. After 4 [cm] the intensity is:

$$\frac{I_x}{I_0} = \exp\left(-\frac{5 \cdot 4}{4.343}\right) = 0.01$$

Only 1% of the intensity reaches the blood clot. The reflection depends on the impedance;
the reflection for liver/blood boundary yields:

$$R_p = \frac{I_{ref}}{I_{in}} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \approx 0.01$$

$$\frac{I_{echo}}{I_0} = \frac{I_x}{I_0} \cdot R_p \cdot \frac{I_x}{I_{ref}} = 0.01 \cdot 0.01 \cdot 0.01 = 10^{-6}$$

Only 1% of the incoming wave is reflected, and only 1% of the reflected wave travels back to the receiver. This means that only 1/1000000 of the initial intensity returns. This requires extremely sensitive detection systems.

If a echo signal originates at a blood clot at only 2 cm depth the intensity reduction is:

$$\frac{I_x}{I_0} = \exp\left(-\frac{5 \cdot 2}{4.343}\right) = 0.1$$

The total loss is due to the attenuation and the reflection probability:

$$\frac{I_{echo}}{I_0} = \frac{I_x}{I_0} \cdot R_p \cdot \frac{I_x}{I_{ref}} = 0.1 \cdot 0.01 \cdot 0.1 = 10^{-4}$$

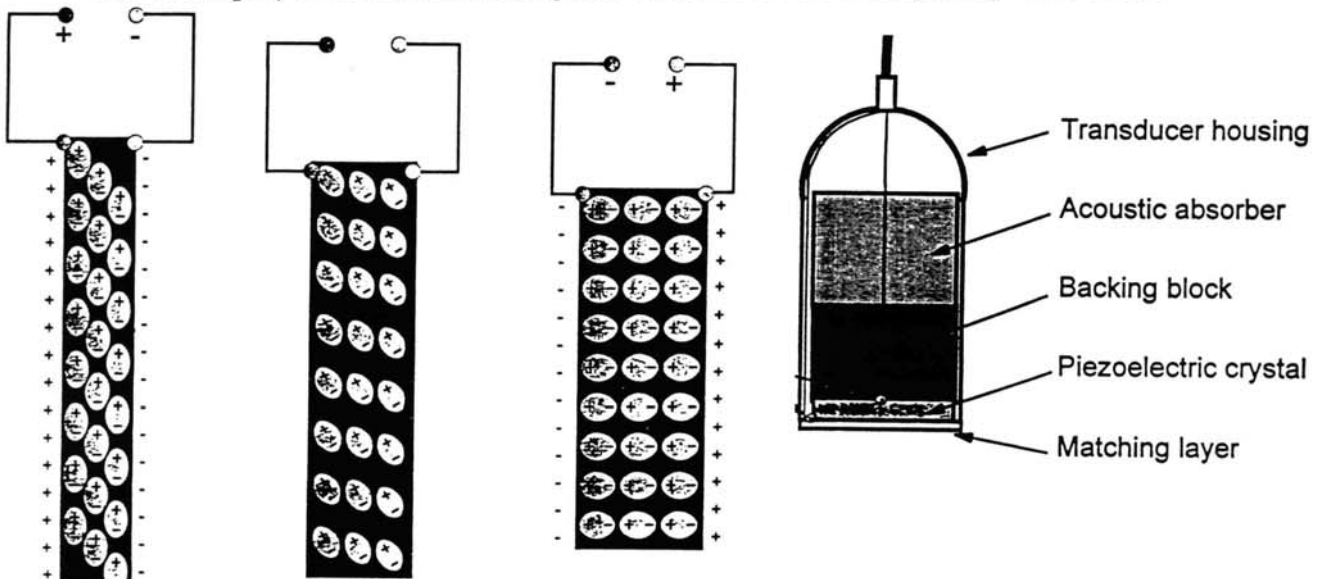
The echo intensity decreases rapidly with depth!

CHARACTERISTICS OF THE ACOUSTIC RADIATION FIELD

The ultrasound signal is generated and detected by the transducer. An electrical pulse causes a change of the thickness t in a piezoelectric element (PZT lead zirconate titanate) which initiates a pressure wave of high frequency f :

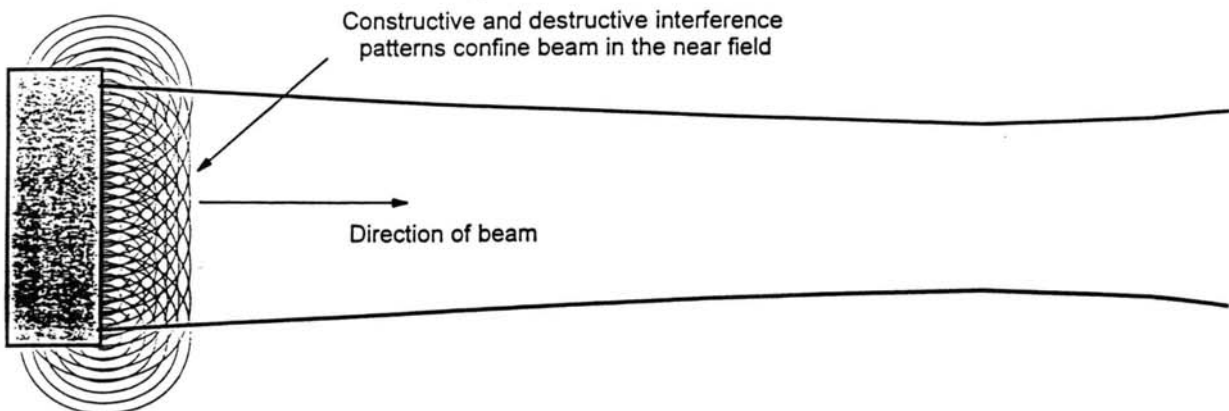
$$f [MHz] \approx \frac{2}{t [mm]}$$

For example, a 0.4 mm thick crystal resonates at a frequency of 5 MHz.

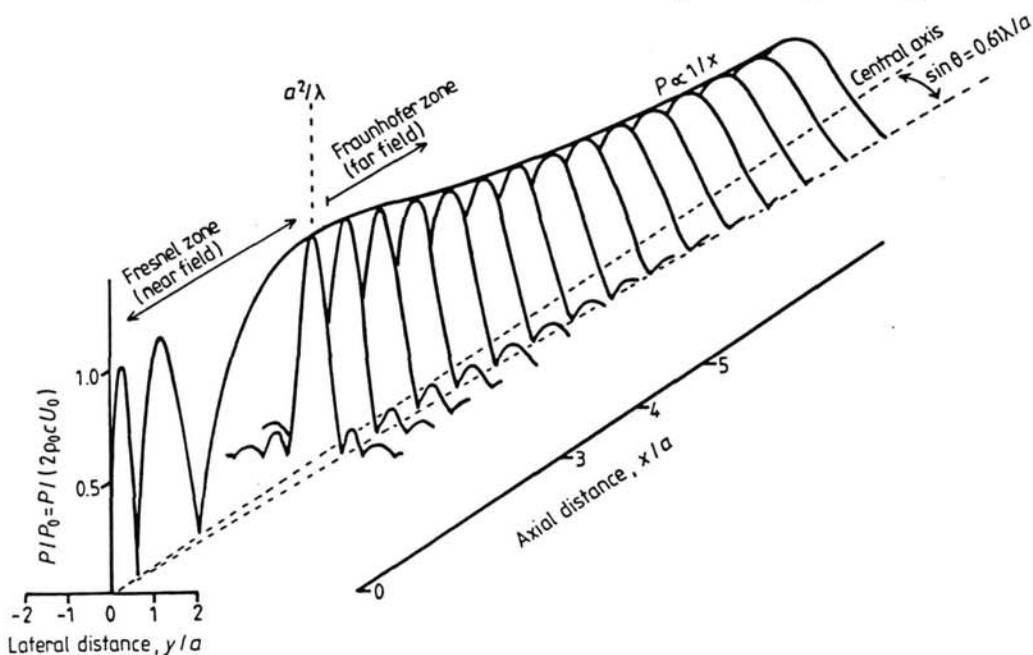


On the other hand, a received high frequency acoustic wave triggers an electrical signal in the two attached electrodes which can be processed.

The sound wave is emitted from the transducer as an overlay of a multitude of single wavelets which form a wave front.



The acoustic beam profile is characterized by the near field and the far field. The pulse in the near zone is characterized by large amplitude variations but the energy is confined within the transducer radius along the axis. In the far zone the field is uniform but spherically divergent.



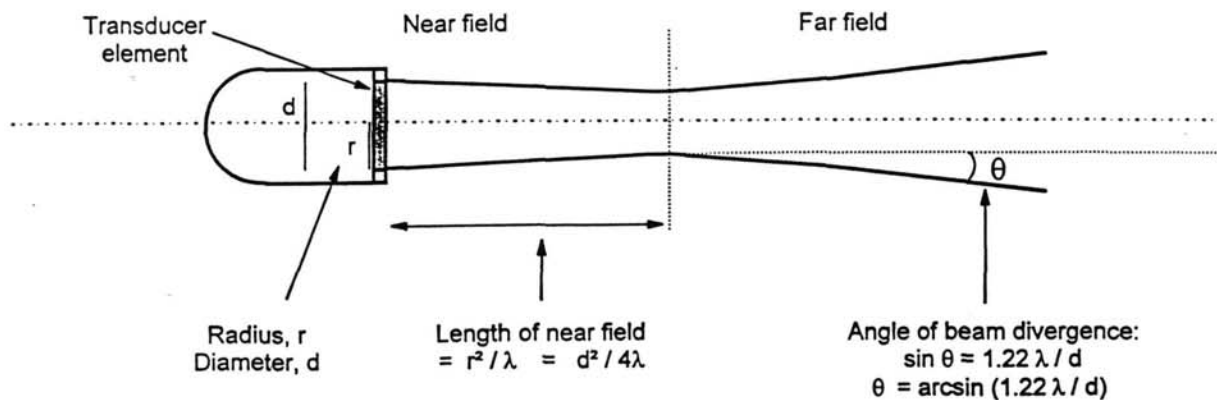
The length of the near field L depends on the frequency and the diameter of the transducer,

$$L = \frac{d^2}{4 \cdot \lambda} = \frac{f \cdot r^2}{v}$$

large radius and frequency cause an extended near zone.

The divergence in the far field is given by the divergence angle Θ

$$\sin \Theta = 1.22 \frac{\lambda}{d}$$

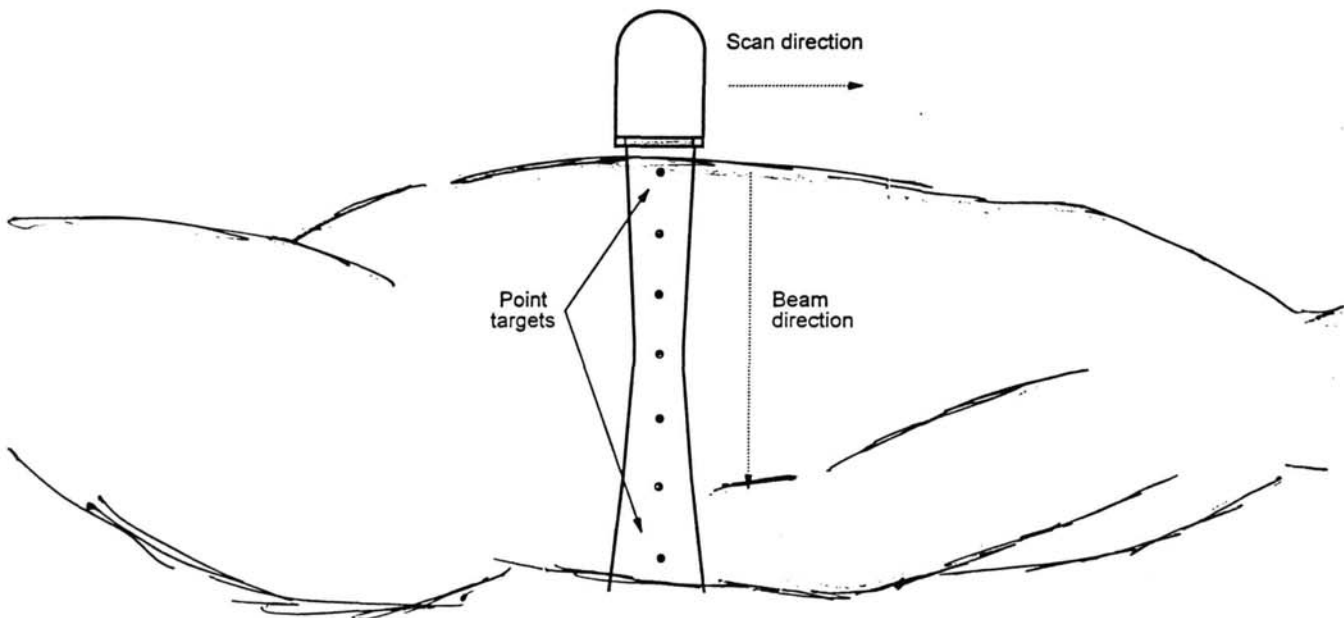


EXAMPLE

A typical PZT-5A piezo-crystal of 2 cm diameter is used to produce an ultrasound frequency of 5 MHz. This allows to calculate the length of the near region:

$$L = \frac{f \cdot r^2}{v} = \frac{5 \cdot 10^6 \text{ [1/s]} \cdot 10^{-4} \text{ [m}^2\text{]}}{1540 \text{ [m/s]}} = 0.325 \text{ [m]}$$

The near range covers nearly the body thickness.



The diverging angle of the far field of the ultrasound beam is:

$$\sin\Theta = 1.22 \frac{v}{f \cdot d} = 0.61 \frac{1540 \text{ [m/s]}}{5 \cdot 10^6 \text{ [1/s]} \cdot 0.1 \text{ [m]}} = 0.002 \rightarrow \Theta = 0.11^\circ$$

Only small divergence takes place in the far region.

Low frequency signals reduce the size of the near range and increase the divergence of the beam in the far range.

PULSE-ECHO-TECHNIQUES

The simple pulse-emission echo-detection technique is the most common application for ultrasound. It relies on the ability of detection of weak echo signals which originate from relatively low intensity pulses (intensity loss causes unnecessary heating of the body organs).

The quality of the signal is determined by its resolution! For pulse echo systems both axial resolution (along the beam axis) and lateral resolution (perpendicular to beam axis) have to be considered. The axial resolution R_{axial} is defined in terms of the half-width δt_{FWHM} of the ultrasound pulse:



$$R_{axial} = \delta t_{FWHM} \cdot v$$

For micro-second pulses (MHz) the typical half widths of the pulse range from 2 mm to 7 mm depending on the kind of body tissue.

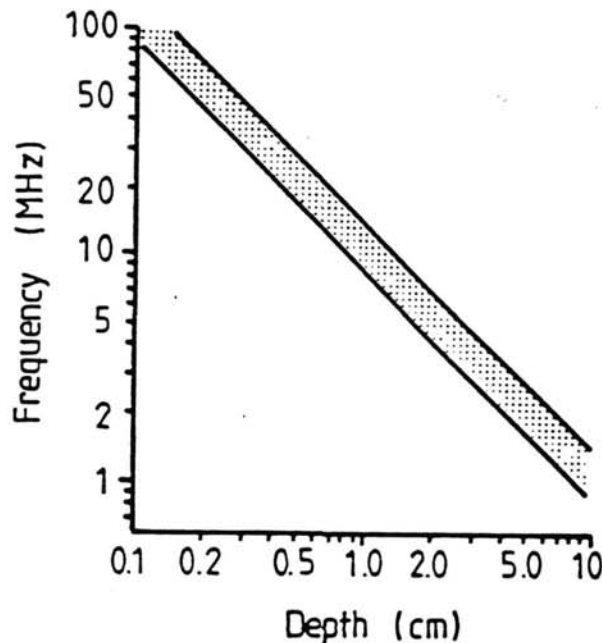
The lateral resolution is determined by the size of the acoustic beam, which depends on the diameter of the transducer element d , the length of the near region and the divergence of the far range.

$$R_{lateral} \approx \delta d_{FWHM}$$

Typically the lateral resolution ranges from 1.0 to 2.0 cm.

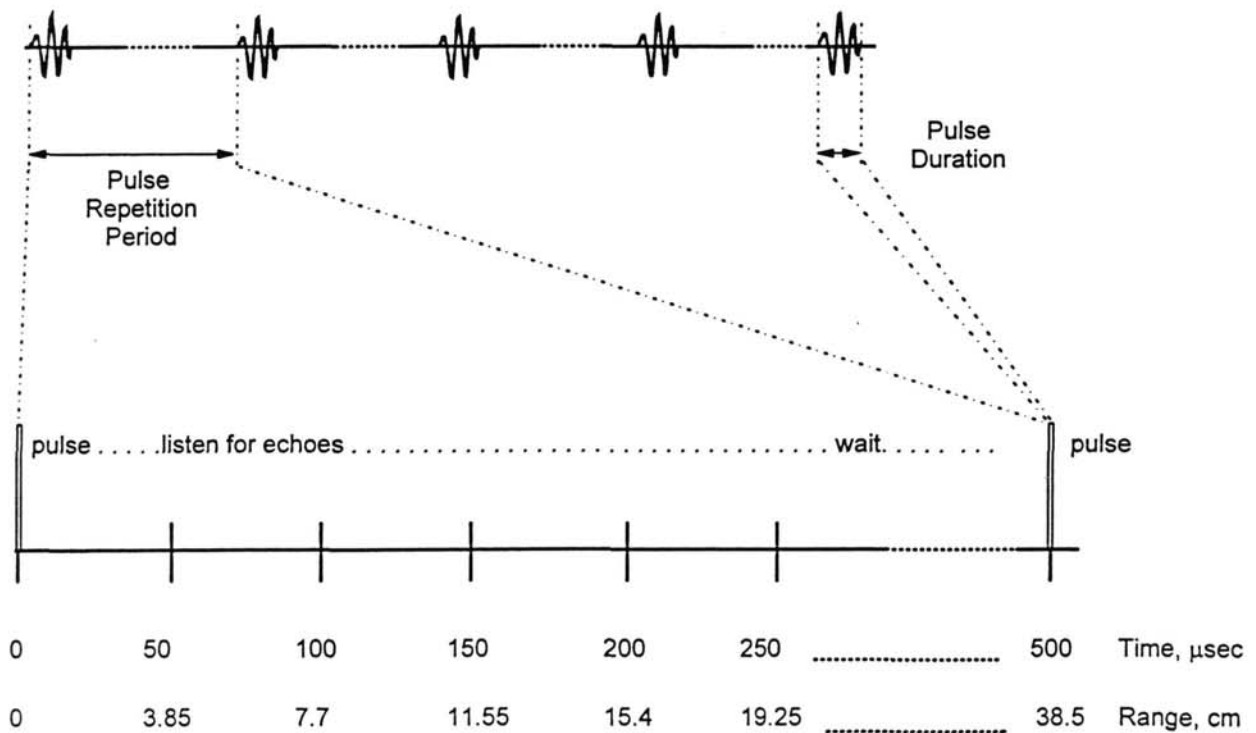
The choice of the ultrasound frequency is determined by a compromise between good resolution and deep penetration. The attenuation of high frequency acoustic waves limits the penetration depth, low frequency waves decrease both axial and lateral resolution.

Frequencies in the 3 - 5 MHz range are used for liver, heart and abdominal organs, higher frequencies in the 4 - 10 MHz range with better resolution are used to probe smaller structures, like thyroid, arteries infant organs etc.



The loss of intensity of the acoustic signal and the returning echo signal might reduce the echo to intensities below the 'noise' level. The noise originates from scattered sounds and from the electronics.

The transducer emits short acoustic signals of ≈ 3 cycles length (for 5 MHz, signal length would be $\delta t \approx 3/5 \cdot 10^{-6} = 0.6 \mu s$) with a certain frequency (pulse repetition rate PRR).



The period between the acoustic pulses is determined by the time the echo needs to return from the maximum distance D_{max} to the transducer. The maximum repetition rate PRR_{max} would be:

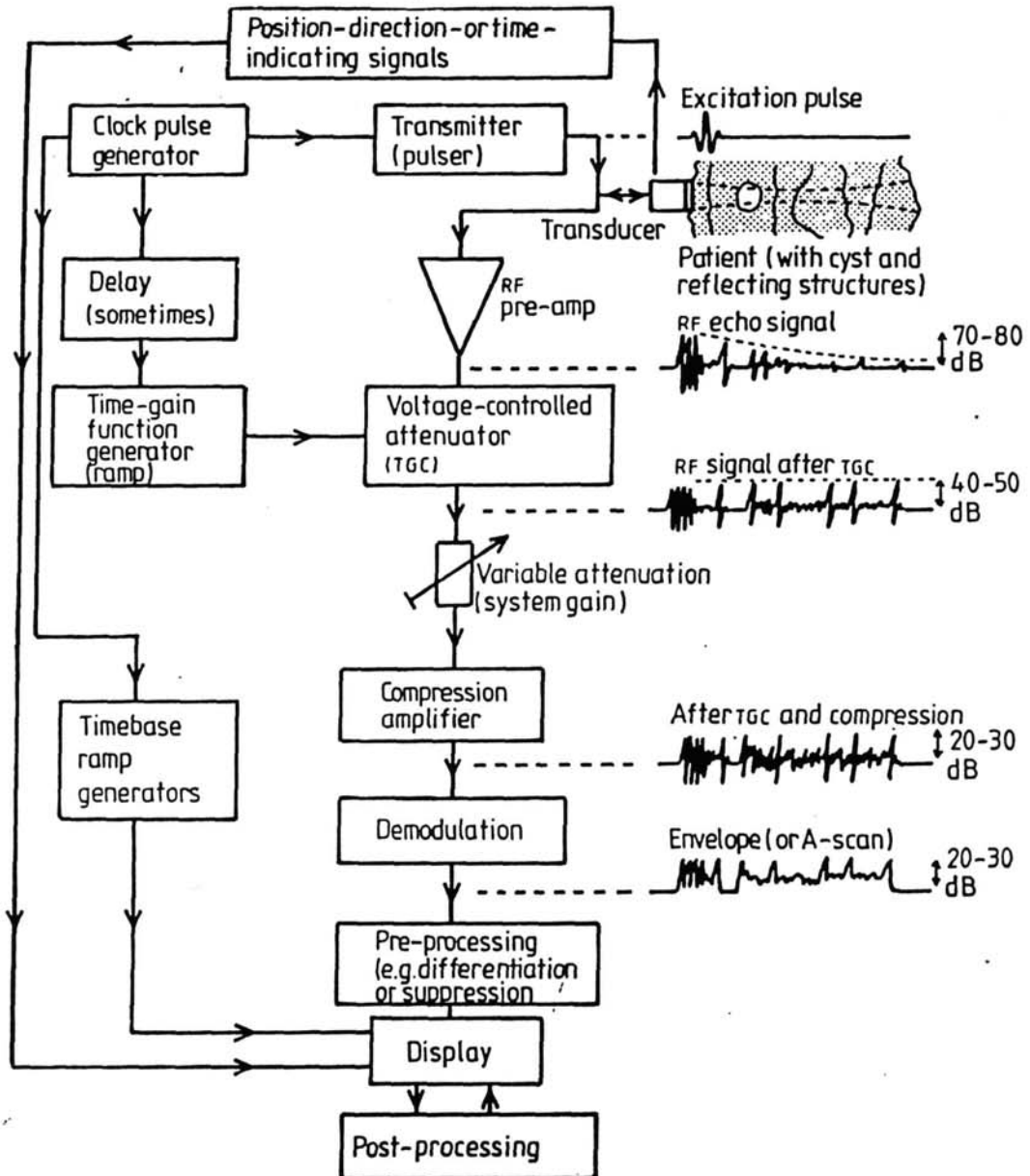
$$PRR_{max} = \frac{1540 [m/s]}{2 \cdot D_{max} [m]}$$

If the repetition rate is exceeded the echo signals become ambiguous because echos from deep layers triggered by the first pulse overlap with echos from surface layers initiated by the following pulse.

Probing organs at a depth of ≈ 20 cm requires a repetition rate of ≈ 600 Hz, typically the repetition rate is in the order of 1 KHz.

SIGNAL-PROCESSING

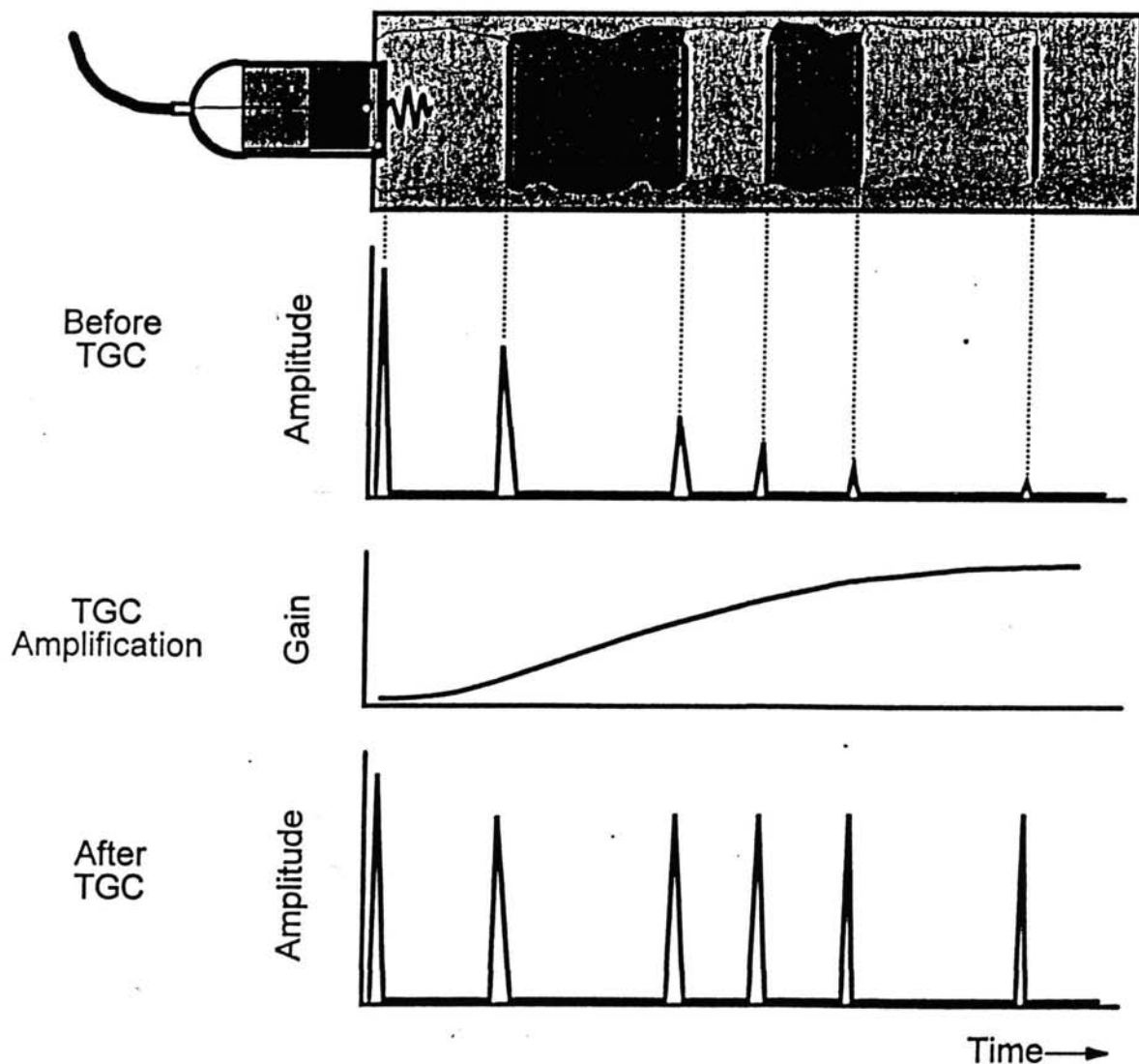
For producing an image the acoustical signal has to be emitted with a constant *PRR* at a constant *db* level (≈ 100 mW), the returning weak echo system has to be amplified to a constant *db* level and electronically processed.



After standard amplification of echo, the most important part is the time gain control (TGC).

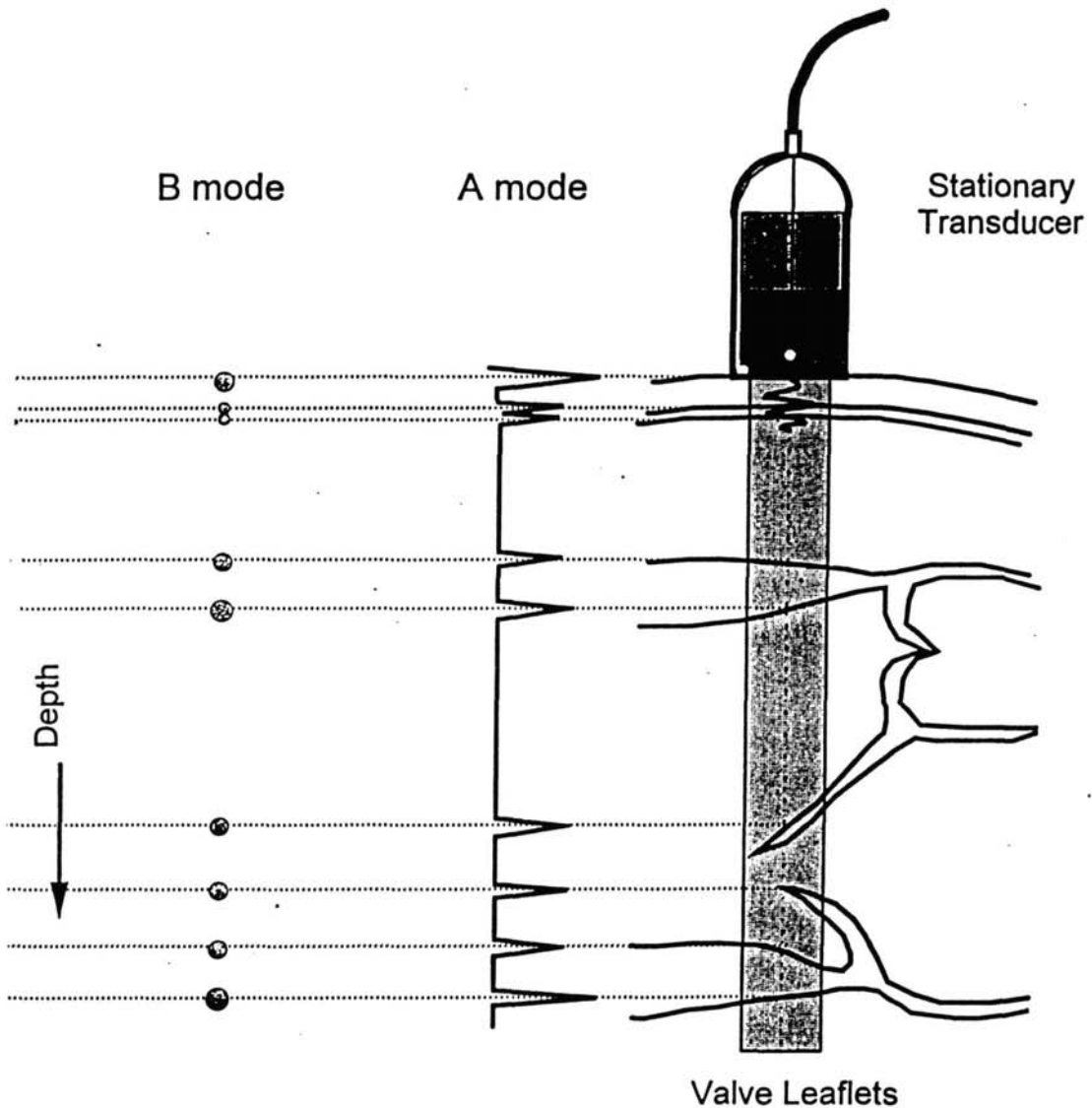
The TGC selectively amplifies echo signals which return weak from a deeper depth at a later time to the level of earlier echo signals from less deep layers which have experienced less attenuation.

Equally reflective acoustic impedance boundaries



Overall signal intensity is then $\approx 40-50$ db.

The signals will be digitized by Analog to Digital Converter (ADC) systems and processed in the computer. The data can be displayed in a one dimensional way as intensity versus time (A-scan), which corresponds to a simple axial scan.



The most common display is the B-scan, where A-scan is coupled with x-y movement of the transducer.

The intensity of the echo signals is translated into brightness of display for different x-y positions, this is used to create the two-dimensional image from the intensity as function of x,y-position $g(x, y)$.

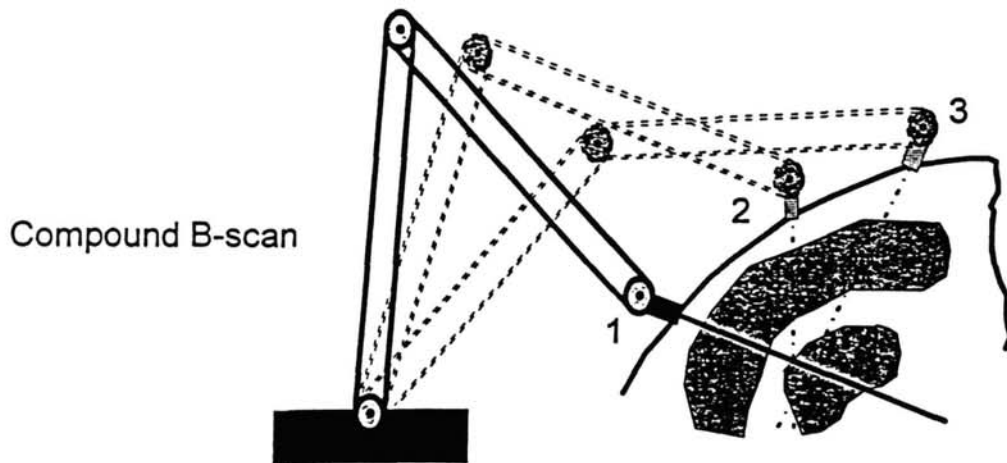
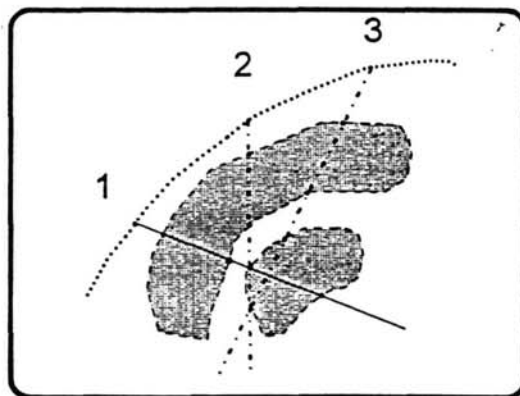


Image display



The construction of the actual image (in the computer) is a complex mathematical process which takes into account the pulse shape characteristics and the density distribution of the body tissue as reflected by the interaction processes between acoustic wave and body material.

THE GENERATION AND CONSTRUCTION OF IMAGES

The structure of the reflected echo signal is a convolution of the pulse-echo wave and the response of the body through signal scattering.

For a two-dimensional description image function $g(x, y)$ can be expressed in terms of the

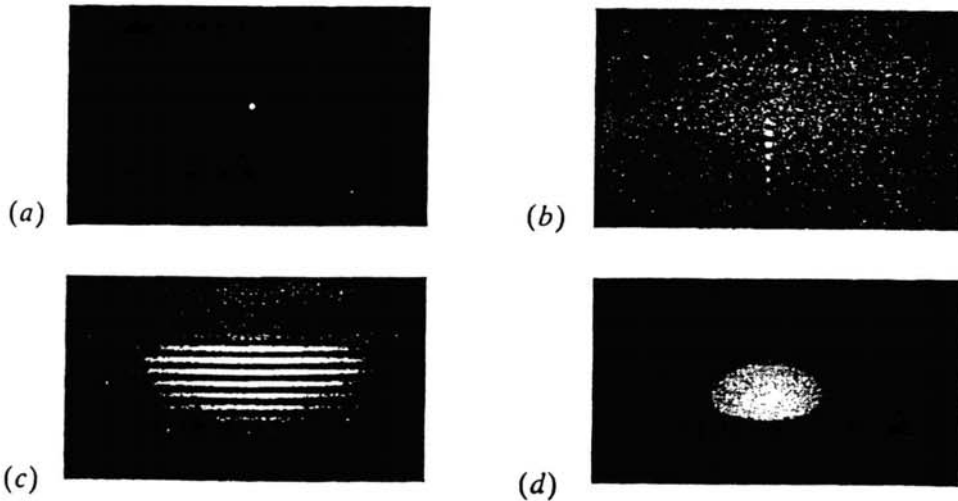
- axial pulse-echo response $h_1(x)$ (pulse shape of signal),
- the lateral pulse-echo response $h_2(y)$ (beam profile),
- the scattering response $f(x, y)$ of the system

$$g(x, y) = h_1(x) \cdot h_2(y) \cdot f(x, y)$$

Very simplified form, neglecting noise, multiple scattering, differences in speed of sound ...

The scattering response depends on the density of material $\rho(x, y)$ and the elastic modulus $K(x, y)$ which determines speed of sound and impedance. The response can be formulated in terms of fluctuations in density and elasticity in the direction of the pulse x

$$f(x, y) = \frac{1}{4} \frac{\partial^2}{\partial x^2} \left(\frac{\delta\rho(x, y)}{\rho_0} + \frac{\delta K(x, y)}{K_0} \right)$$



The simulated image of a point scatterer convoluted with the time structure of the pulse echo signal and the extension of the acoustic beam.