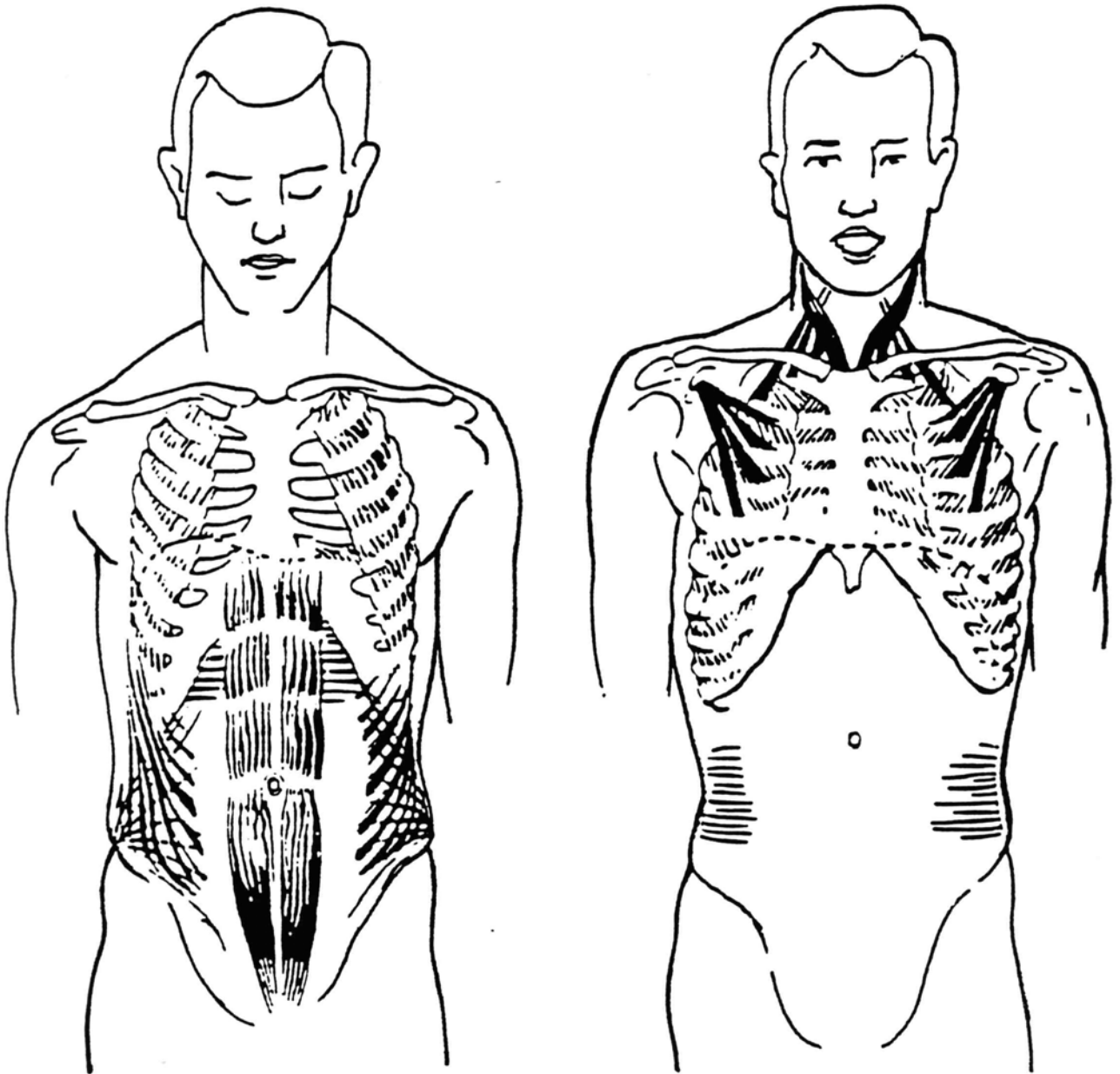


3. THE PRESSURE SYSTEM OF THE BODY

3.1. THE PHYSICS OF BREATHING



Units of Pressure

pressure P is defined by the amount of force F on a certain area A :

$$P \equiv \frac{F}{A}$$

The international standard unit for pressure is the *Pascal*:

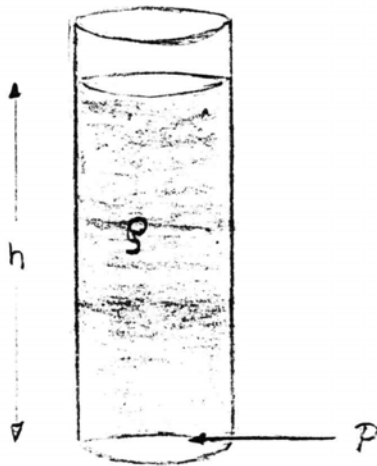
$$1 \text{ [Pa]} \equiv 1 \text{ [N/m}^2\text{]}$$

typical examples:

$$1 \text{ atmosphere: } 1 \text{ atm} = 1.01 \cdot 10^5 \text{ Pa}$$

$$\text{bicycle tire: } = 414 - 620 \text{ kPa} \approx 60 - 90 \text{ lb/in}^2 \text{ (psi)}$$

Traditional unit for pressure in the medical community is the mm Hg which corresponds to the pressure at the bottom of a column of mercury Hg of a certain height h :



$$P = \rho \cdot g \cdot h$$

ρ is density of liquid

$g = 9.81 \text{ m/s}^2$ earth acceleration

With a density of Hg, $\rho=13.6 \cdot 10^3 \text{ kg/m}^3$, a column of Hg of $h=1\text{mm}$ causes a pressure of:

$$P=133 \text{ Pa} \equiv 1 \text{ mm Hg}$$

for water with a density pf $\rho=10^3 \text{ kg/m}^3$ the pressure is smaller by approximately a factor of 13.6:

$$P=9.81 \text{ Pa} \equiv 1 \text{ mm H}_2\text{O} = 0.1 \text{ cm H}_2\text{O} = 0.0735 \text{ mm Hg}$$

Some of the Common Units Used to Measure Pressure

	Atmospheres	Pa	cm H ₂ O	mm Hg	lb/in ² (psi)
1 atmosphere	1	1.01×10^5	1033	760	14.7
1 Pa	0.987×10^{-5}	1	0.0102	0.0075	0.145×10^{-3}
1 cm H ₂ O	9.68×10^{-4}	98.1	1	0.735	0.014
1 mm Hg	0.00132	133	1.36	1	0.0193
1 lb/in ² (psi)	0.0680	6895	70.3	51.7	1

Typical blood pressure for healthy adult ranges between
 $P_{blood} \approx 100 - 120 \text{ mm Hg}$ (13.3 - 15.8 kPa)

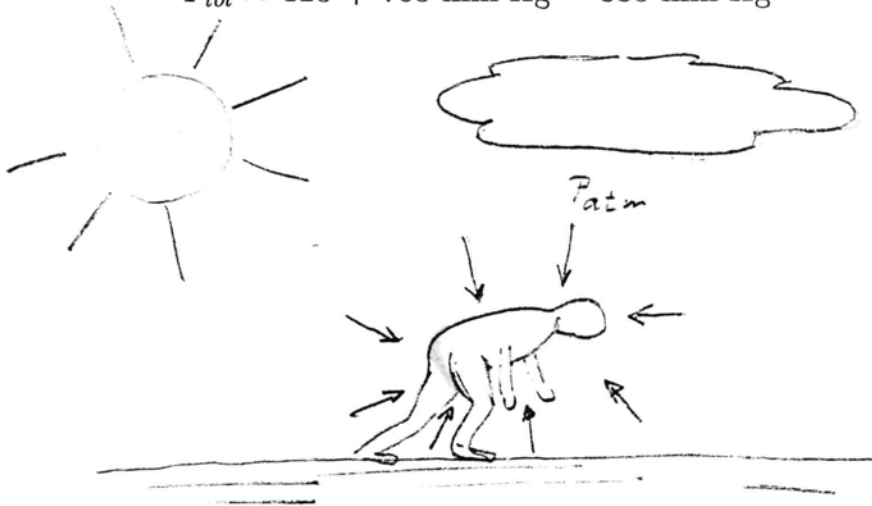
However, man exists at certain conditions of outer environment, his internal pressure system is therefore influenced by the outside pressure conditions. The total pressure is defined by the sum of the external pressure plus the internal pressure (gauge pressure).

EXAMPLES PRESSURE CONDITIONS OF TYPICAL OUTER ENVIRONMENTS

the typical air pressure on the entire body is 1 atm = 760 mm Hg;

the total blood pressure is therefore:

$$P_{tot} \approx 120 + 760 \text{ mm Hg} = 880 \text{ mm Hg}$$



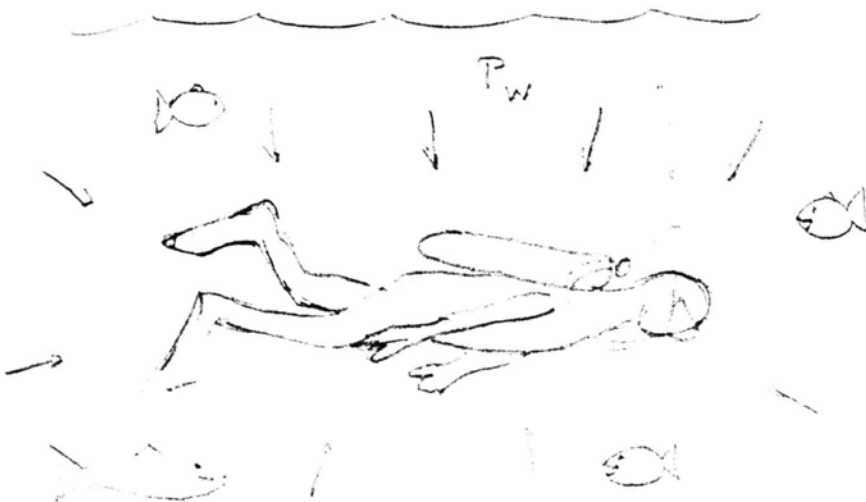
for a diver at a water depth of 30 m the external pressure is:

$$P_w = 10^3 \text{ kg/m}^3 \cdot 9.81 \text{ m/s}^2 \cdot 30 \text{ m} = 284 \text{ kPa} = 2210 \text{ mm Hg}$$

considerably larger than air pressure

The total blood pressure is therefore:

$$P_{tot} \approx 120 + 2210 \text{ mm Hg} = 2330 \text{ mm Hg}$$



Typically the quoted pressure corresponds to the gauge pressure!

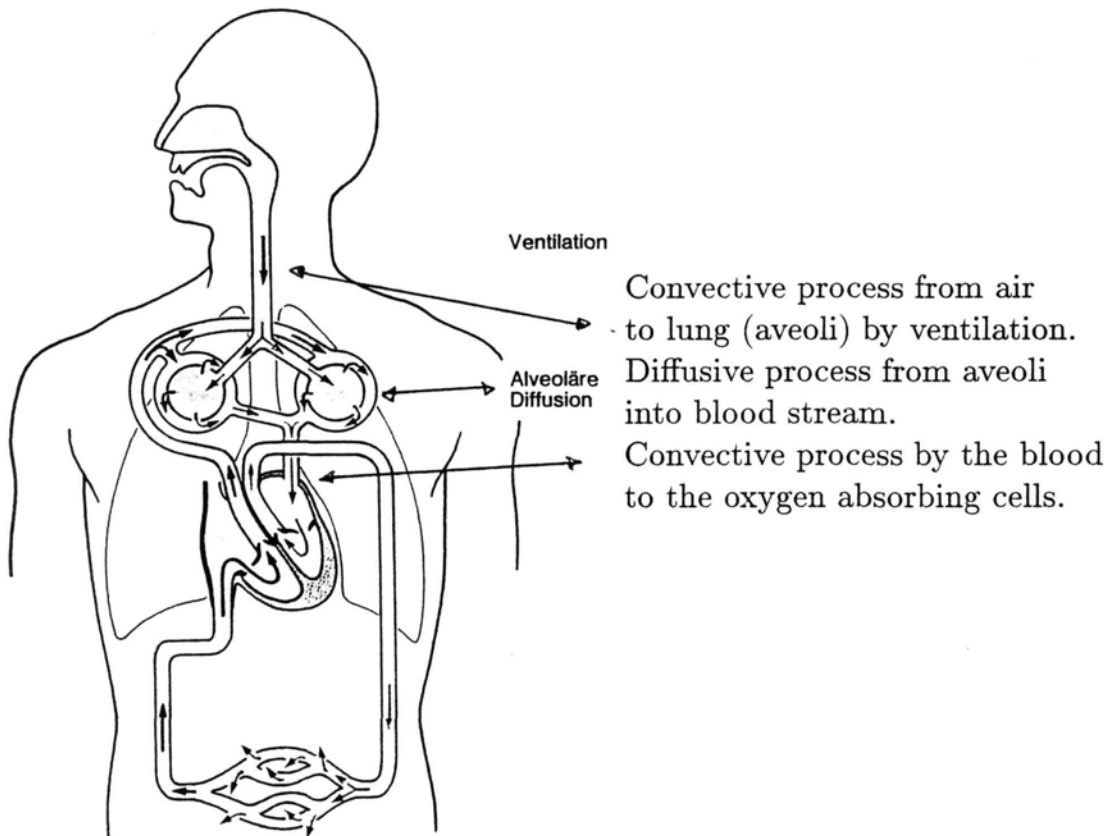
Typical Pressures in the Normal Body

	Typical Pressure	
	kPa	(mm Hg)
Arterial blood pressure		
Maximum (systole)	13-18	100-140
Minimum (diastole)	8-12	60-90
Venous blood pressure	0.4-0.9	3-7
Great veins	<0.1	<1
Capillary blood pressure		
Arterial end	4	30
Venous end	1.3	10
Middle ear pressure	<0.1	<1
Eye pressure—aqueous humor	2.6	20
Cerebrospinal fluid pressure		
in brain (lying down)	0.6-1.6	5-12
Gastrointestinal	1.3-2.6	10-20
Intrathoracic pressure		
(between lung and chest wall)	-1.3	-10

The various listed pressure conditions will be discussed in the following sections.

Gas Transport in the Respiratory System

Main purpose of the respiratory system is to transport oxygen O_2 from the outer environment into the blood stream and supply the body with the necessary fuel for the oxidization processes. Second purpose is to remove the rest product of the oxidization process CO_2 from the blood and exhale it to the outer environment. Different transport mechanisms are utilized for this purpose, however, all these transport mechanisms are based on pressure differences in the respiratory system.



The transport of the CO_2 takes place by identical processes in reverse direction. Both processes take place at the same time, therefore pressure differences have to be adopted to optimize the transport mechanism. The inhaled air is a gas mixture of $\approx 80\% N_2$ and $20\% O_2$. The exhaled air is $\approx 80\% N_2$, $16\% O_2$, and $4\% CO_2$. $\Rightarrow$ 4% oxygen is absorbed and exchanged with carbon dioxide.

The average human being breathes ≈ 6 l/min of air

$$1 \text{ mol (air)} = 0.8 \cdot 28 \text{ g (N}_2\text{)} + 0.2 \cdot 32 \text{ g (O}_2\text{)} = 28.8 \text{ g}$$

$$1 \text{ mol} \equiv 22.4 \text{ l}$$

$$\Rightarrow \text{breathing rate} \approx 8 \text{ g/min} \approx 11 \text{ kg/day}$$

$$1 \text{ mol} \equiv 6.022 \cdot 10^{23} \text{ molecules}$$

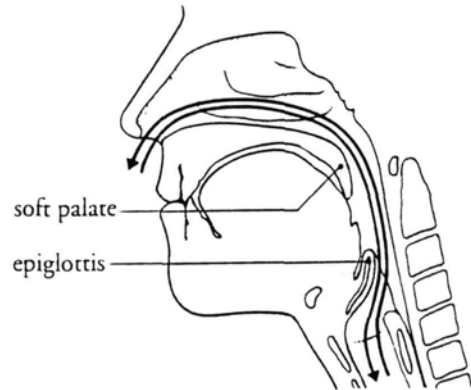
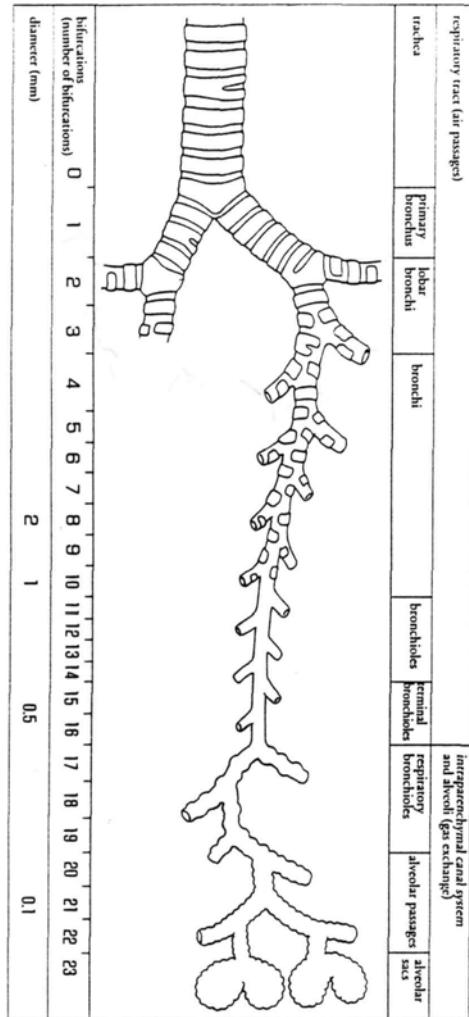
$$\Rightarrow \text{breathing rate} \approx 1.6 \cdot 10^{23} \text{ molecules/min}$$

men	≈ 12 breaths/min	$\approx 1.3 \cdot 10^{22}$ molecules/breath
women	≈ 20 breaths/min	$\approx 8 \cdot 10^{21}$ molecules/breath
children	≈ 60 breaths/min	$\approx 3 \cdot 10^{21}$ molecules/breath

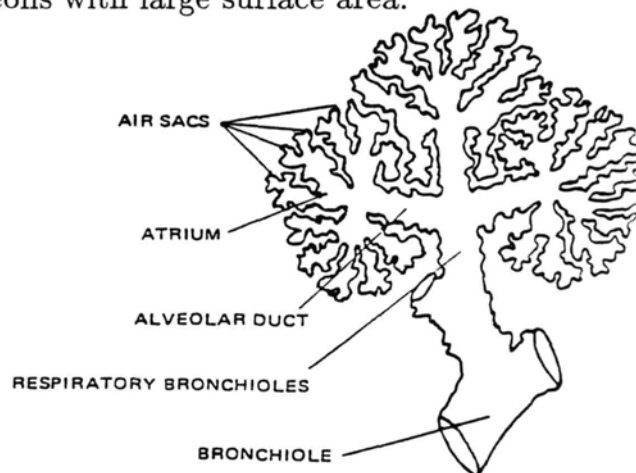
$$\text{total number of molecules in atmosphere: } \approx 10^{44}$$

$$\Rightarrow \text{with each breath we inhale } 10^{-20} \% \text{ of the atmosphere}$$

The airways are divided into several sections, to filter, clean and distribute the air.



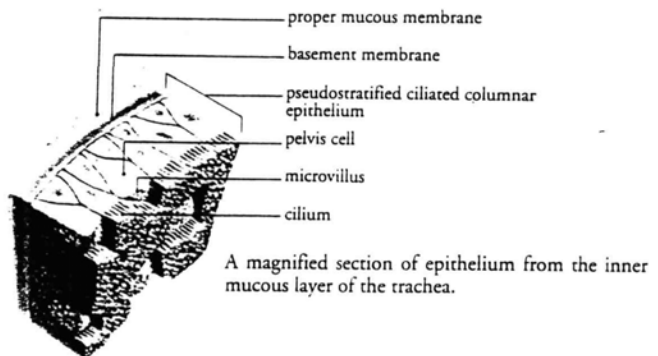
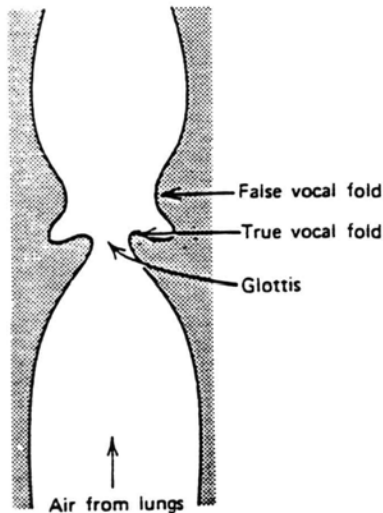
nose: warms and filters the air
mouth: bypass for rapid breathing
trachea: windpipe divides into two
bronchia: subdivides fifteen times
(for optimal distribution)
into bronchioles terminated by
aveolis with large surface area.



With multiplying subdivision of air passages the cross section of transport system increases rapidly. Convective movement slows down, diffusive processes take over.

Air passage has also cleaning purposes. Three cleaning mechanisms:

- **nose** filters through fine hair dust particles from the inhaled air.
- **caughing** reflectory increase of pressure in trachea against closed glottis (opening between vocal folds) and ejection of dust and larger particles by sudden opening of glottis.
- **cilia motion** transport of smaller dust particles by wave motion of cilia hair upwards.



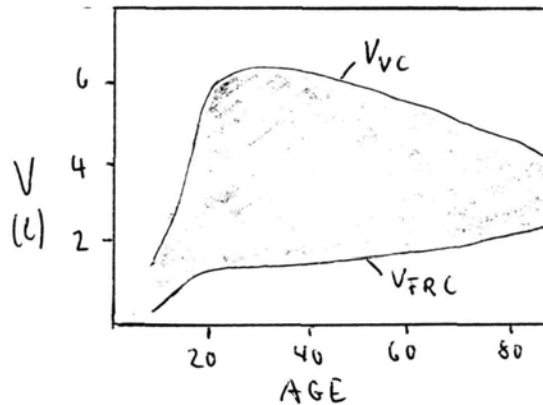
temperature aspects of the respiratory system

- warming the inhaled air up to body temperature 37°
- removing body heat by exhaling warm gases.

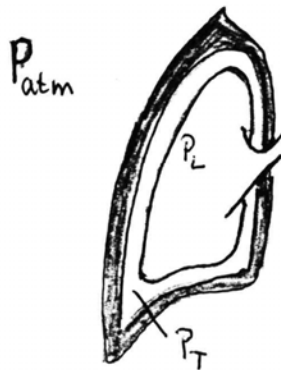
Definition of Pressures and Volumes in the Lung-Thorax System

Volume of vital capacity	V_{VC} maximum volume exhaled after maximum
Restvol. of max. expiration	V_{FRC} remaining volume after maximum expiratic
Tidal volume	V_T average volume exhaled (or inhaled)

$$V_{VC} \approx 2.5 \cdot \text{height [m]} \text{ (in units liter)}$$

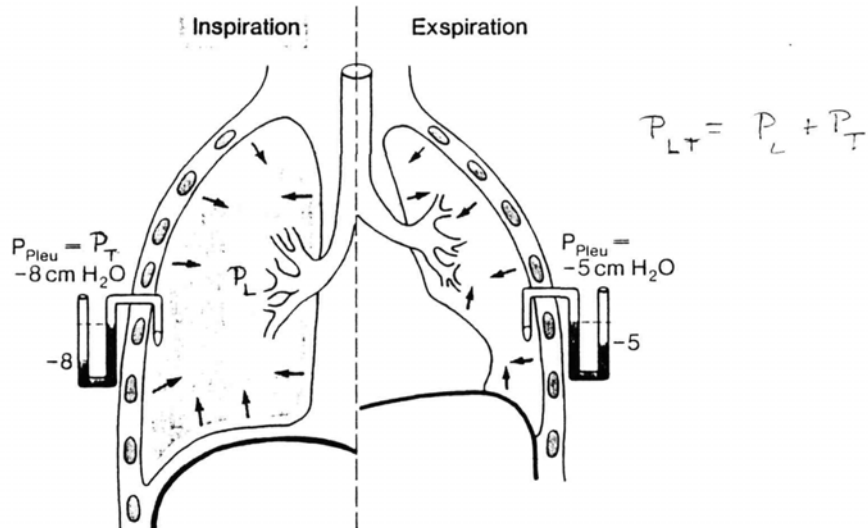


pressure in thorax	$P_T = P_{thorax} - P_{atm}$	pressure often negative!
pressure in lung (aveoli)	$P_L = P_{lung} - P_{atm}$	positive if inhaled, ≈ 0 if exhaled
pressure in lung-thorax	$P_{LT} = P_L + P_T$	time dependent function



Pressure and Volume of the Lungs

The breathing - inhaling and exhaling of gases - is controlled by the pressure volume conditions in the lungs.

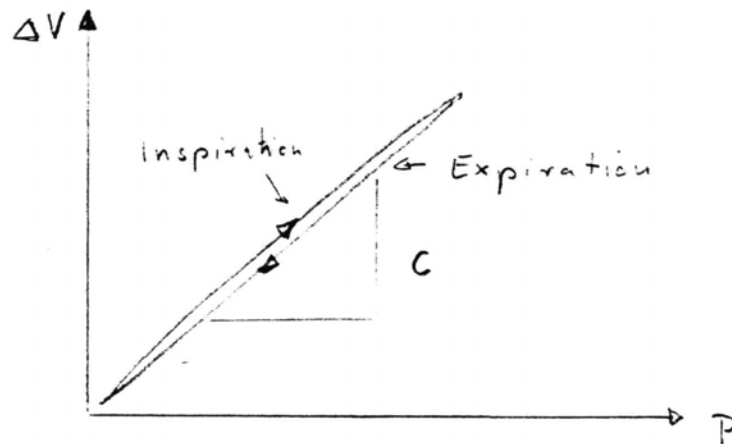


Lungs and chest wall are coupled together, lungs are sitting inside the chest in an airtight system. Pressure between lungs and chest wall is called intrapleural or intrathoracic pressure, typically negative with respect to pressure inside the lung.

Pressure changes are coupled by the change of the volume of the lung. The relation is nearly linear and is limited by the elasticity of the lung tissue. A measure of the elasticity of the respiratory system is the *compliance*. The compliance is determined by the ability of the pressure in the lung system of changing the volume:

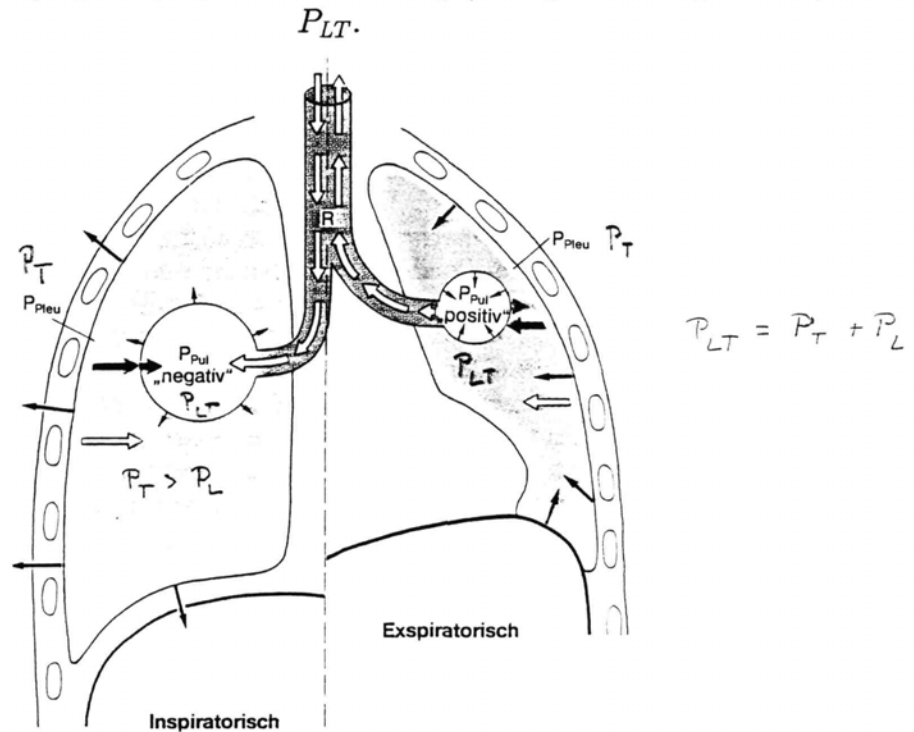
$$C \equiv \frac{d(\Delta V)}{dP}$$

The compliance corresponds to the slope of the inhale-exhale curve in the pressure-volume diagram:



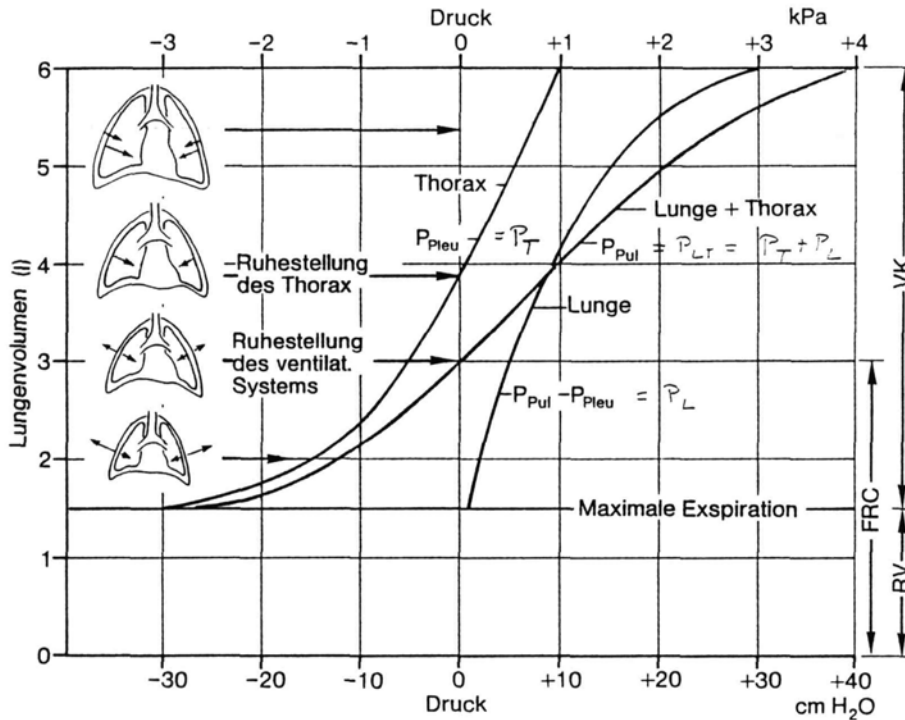
$\Delta V \equiv$ change of volume of the lung
 $P \equiv$ gauge pressure in the lung

Because of close coupling between lung and thorax, expansion of chest also influences the pressure conditions in the intrapleural (intrathoracic) space (P_T) and in the lung (intrapulmonal pressure)



Compliance of thorax: $C_T = d\Delta V / dP_T$
 Compliance of lung: $C_L = d\Delta V / dP_L$
 Compliance of lung-thorax system: $C_{LT} = d\Delta V / dP_{LT}$

with $P_L = P_{LT} - P_T$



restvolume at maximum expiration: $FRC \approx 1.5 \text{ l}$

volume of vital capacity: $V_{VC} \approx 5 \text{ l}$

this yields a relation between the compliances:

$$\frac{1}{C_{LT}} = \frac{1}{C_L} + \frac{1}{C_T}$$

typical values for the compliances are:

$$C_{LT} = 0.1 \text{ l/cm H}_2\text{O} = 1 \text{ l/kPa}$$

$$C_L = 0.2 \text{ l/cm H}_2\text{O} = 2 \text{ l/kPa}$$

$$C_T = 0.2 \text{ l/cm H}_2\text{O} = 2 \text{ l/kPa}$$

Resistance for the Air Passage

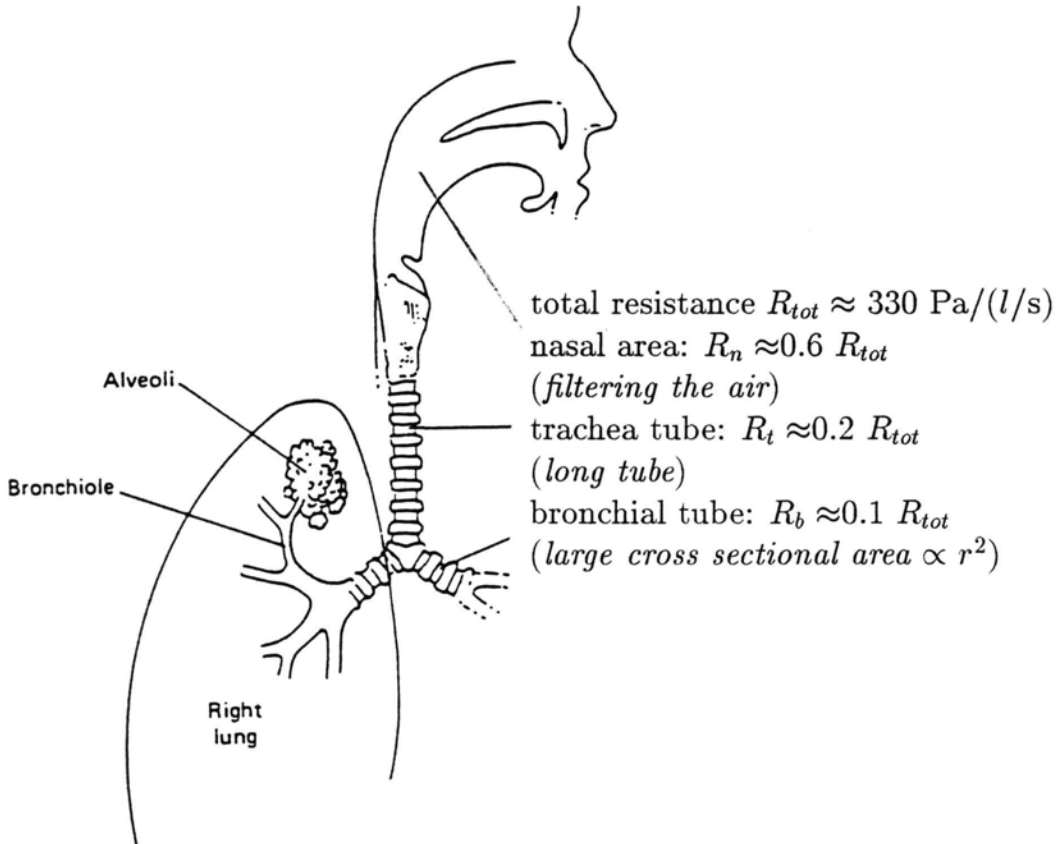
The air flow is proportional to the change of volume per time: $F = dV/dt$

The relation between change of volume and the pressure difference $\Delta P = P_{LT}$ is given by the Hagen Poiseuille law which describes the laminar airflow:

$$F = \frac{dV}{dt} = \frac{\Delta P}{R}$$

R represents the flow resistance which is determined by the viscosity of the fluid η (air), the length L and the radius of the tubes r :

$$R = \frac{8}{\pi} \eta \frac{L}{r^4}$$



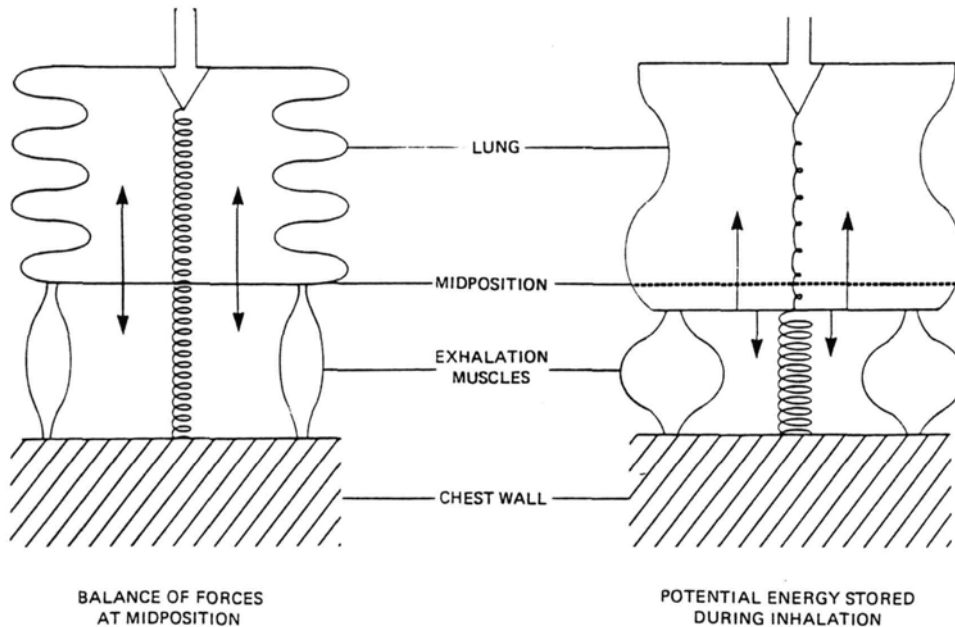
The total resistance is the sum of the single resistances along the air passage

$$R_{tot} = R_n + R_t + R_b$$

Timing for the Breathing Process

similar to oscillatory effects like in the em-RC circuit, or in spring oscillations

The timescale of the breathing process depends on the compliance of the respiratory system and on the resistance for the air flow!



The airflow while inhaling or exhaling, $F = \frac{dV}{dt}$, is determined by the compliances C and the resistances R in the respiratorial system. The airflow can be described than as a function of pressure difference ΔP and the tidal volume $\Delta V_T = V_{VC} - V_{FRC}$

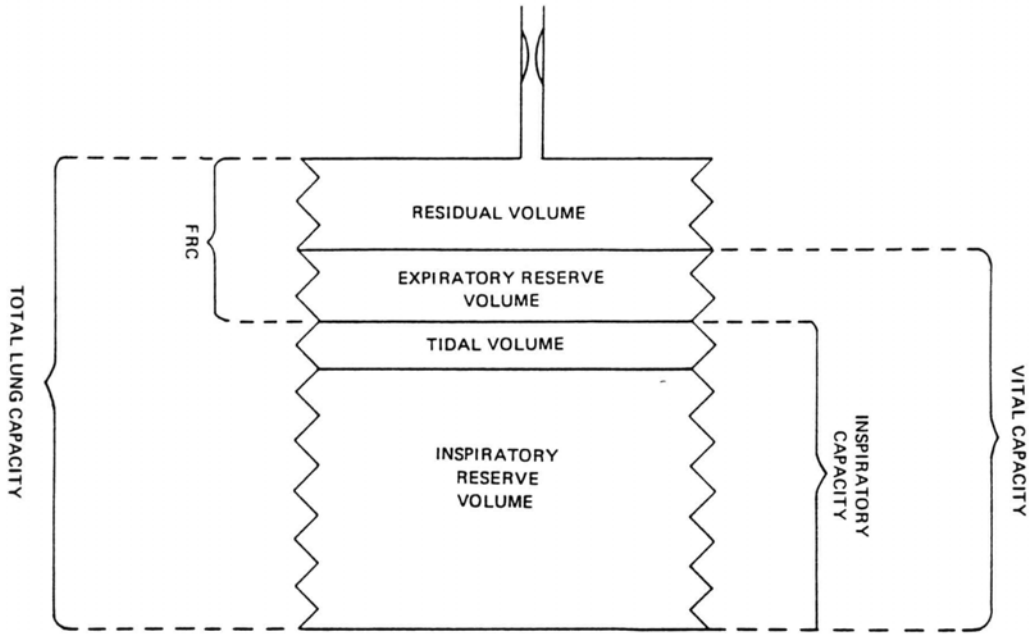
$$\Delta P - F \cdot R - \frac{\Delta V_T}{C} = 0$$

in the units:

$$\frac{N}{m^2} - \frac{l}{s} \frac{Ns}{m^2 l} - \frac{lN}{lm^2}$$

at time $t=0$ lung is at minimum volume, residual capacity (FRC):

tidal volume: $\Delta V_T \approx 0$



Inspiratory capacity and tidal capacity.

$$F = \frac{\Delta P}{R}$$

This corresponds to the maximum flow intake by the lung.

The rate of change is determined by differential equation:

$$\frac{d(\Delta P - F \cdot R - \frac{\Delta V_T}{C})}{dt} = 0$$

$$\frac{d\Delta P}{dt} - \frac{d(F \cdot R)}{dt} - \frac{d\frac{\Delta V_T}{C}}{dt} = 0$$

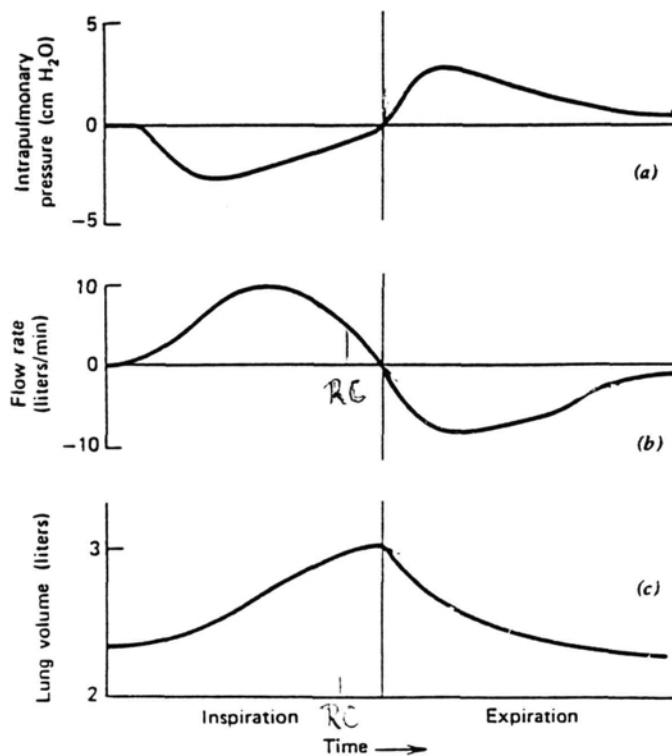
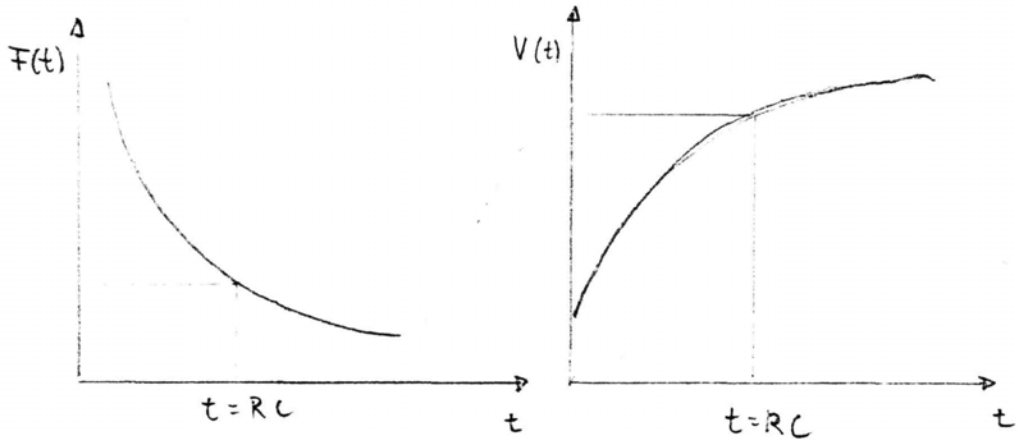
As the intrapulmonal pressure $P_{LT} = \Delta P$ shows only relatively small changes (-1 cm H₂O to + 1 cm H₂O) we can approximate:

$$\frac{d(F \cdot R)}{dt} + \frac{d\frac{\Delta V_T}{C}}{dt} = 0$$

solution of this simple differential equation yields time dependent behaviour for flow (breathing rate) and volume:

$$F(t) = F_{t=0} e^{-\frac{t}{RC}} = \frac{d\Delta P}{R} e^{-\frac{t}{RC}}$$

$$V(t) = C \cdot \Delta P (1 - e^{-\frac{t}{RC}})$$



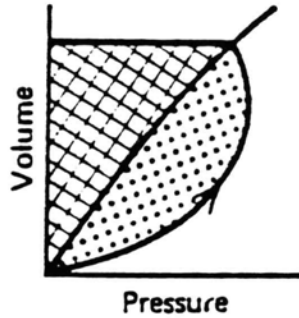
Work required for Breathing

The amount of work ΔW necessary to inhale and exhale is described as function of change of the tidal volume:

$$\Delta V_T = V_{VC} - V_{FRC}$$

$$\Delta W = \int P d\Delta V \quad P = \frac{\Delta V}{C}$$

for inhaling from rest volume of maximum respiration V_{FRC} to maximum volume V_{VC}



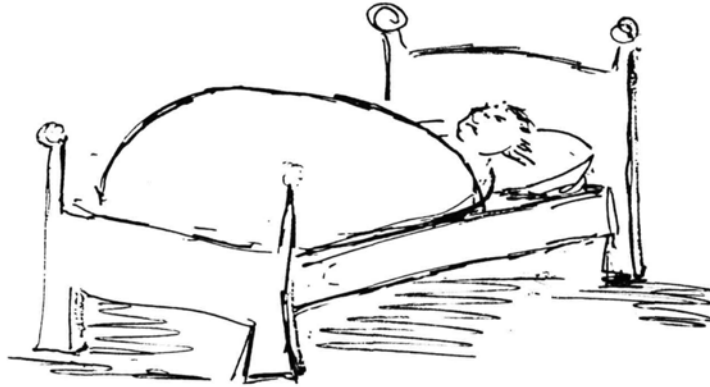
$$\Delta W = \int_0^{\Delta V_T} \frac{\Delta V}{C} d\Delta V$$

$$\Delta W = \frac{1}{2C} \Delta V_T^2$$

The amount of work is directly proportional to the tidal volume.
The rate at which this work is done depends on the number of breaths per time dn/dt

$$\frac{dW}{dt} = \Delta W \cdot \frac{dn}{dt} = \frac{1}{2C} \Delta V_T^2 \cdot \frac{dn}{dt}$$

Example: The sleeping man



With $V_T \approx 2 \text{ l}$, $C \approx 0.1 \text{ l/cmH}_2\text{O}$ $dn/dt \approx 10 \text{ breaths/min}$ (adult)

$$\frac{dW}{dt} = \frac{2^2 l^2}{2 \cdot 0.001 l / Pa} \cdot 10 \text{ min}^{-1} = 20 \frac{\text{Joules}}{\text{min}} = 6.8 \frac{\text{kcal}}{\text{day}}$$

with an average efficiency of $\epsilon \approx 20 \%$ for the conversion of energy into work the total amount of energy used up by the respiratory machine can be calculated:

$$\frac{dU}{dt} = \frac{1}{\epsilon} \frac{dW}{dt} = 34 \frac{\text{kcal}}{\text{day}}$$

This amount corresponds to $\approx 2 \%$ of the total basal rate!

$$\frac{dU}{dt_{bas}} = 1680 \frac{\text{kcal}}{\text{day}}$$

Heat Loss from the Respiratory System

Volume of exhaled air in an average breath:

$$\Delta V_T \approx 0.4 \frac{\text{liter}}{\text{breath}}$$

One breath lasts about:

$$t_{\text{breath}} = 2.5 \cdot C \cdot R = 0.75 \text{ [s]}$$

Compliance: $C \approx 0.001 \text{ [l/Pa]}$ Resistance: $R \approx 300 \text{ [Pa/l/s]}$

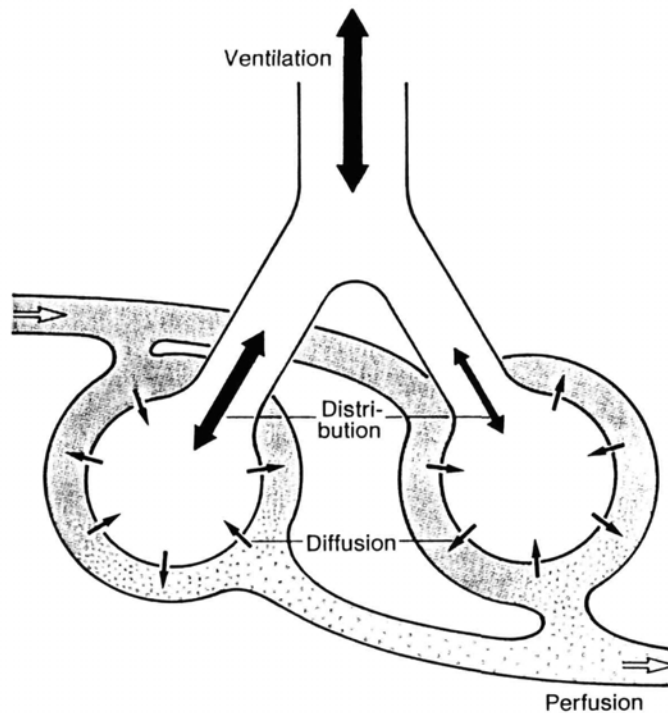
$$\Delta Q = C \cdot (T_L - T_{at})$$

$$T_L = 37^\circ\text{C} \quad T_{at} \approx 21^\circ\text{C} \quad C \approx 7 \cdot 10^{-3} \text{ kcal/mol}^\circ\text{C}$$

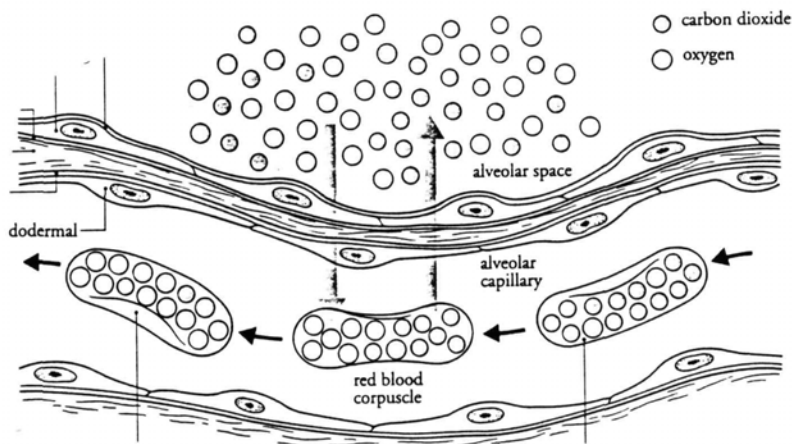
$$\Delta Q = C \cdot (T_L - T_{at}) \cdot \frac{\Delta V}{t_{\text{breath}}} = 2.7 \cdot 10^{-3} \text{ [kcal/s]} = 9.6 \text{ [kcal/h]}$$

Result depends on breathing volume and timing

Diffusive Gas Exchange in the Alveoli



The gas exchange takes place by diffusion through the surface membrane of the aveoli system. The aveoli are small gas bubbles (radius $R \approx 0.2\text{mm}$) at the end of the bronchial system. The lung system has about 30 million (child) to 300 million (adult) aveoli.



Aveoli have a liquid layer between the wall and the air volume.

⇒ surface tension which tends to contract aveoli bubble

$$\text{tension pressure: } P = 4 \gamma / R$$

the tension force coefficient is small: $\gamma \approx 5 \cdot 10^{-5} - 50 \cdot 10^{-5} \text{ N/m}$

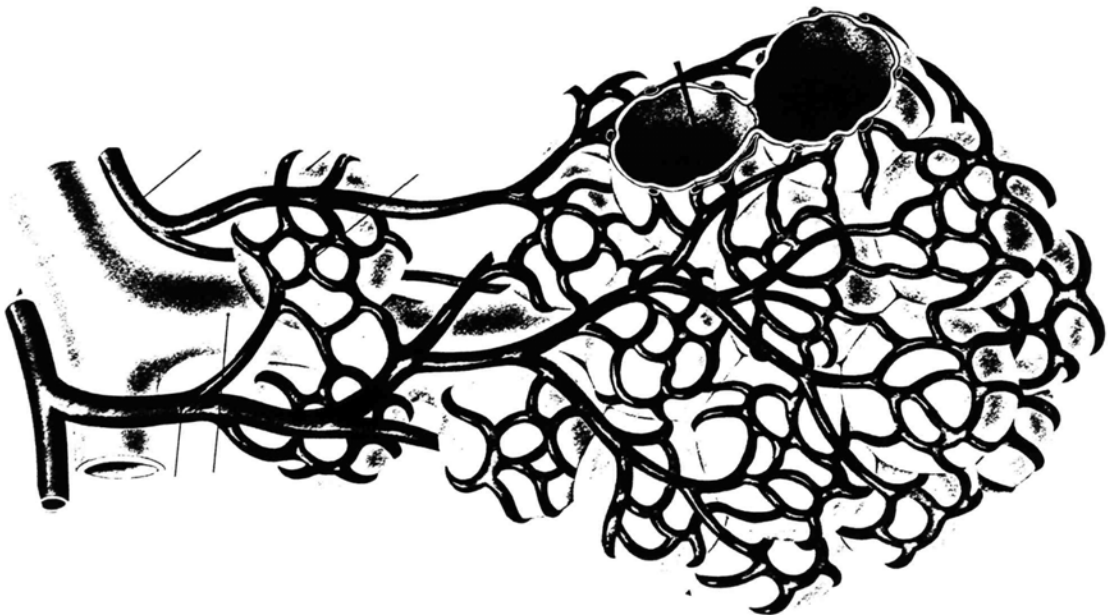
it varies with volume; this prevents the collapse of the aveoli:

$$\text{large volume: } P = 4 \cdot (50 \cdot 10^{-5} / 0.1 \cdot 10^{-3} \approx 20 \text{ Pa})$$

$$\text{small volume: } P = 4 \cdot (5 \cdot 10^{-5} / 0.01 \cdot 10^{-3} \approx 20 \text{ Pa})$$

⇒ Pressure in the aveoli remains approximately constant!

Aveoli are surrounded by capillaries blood vessel system. Separating membrane has a thickness of $\approx 0.1 \mu\text{m}$. The surface area A of the entire aveoli system is $A \approx 80 \text{ m}^2$.



The purpose of the diffusion process is the exchange O_2 with CO_2 . The diffusion flow depends on the size of the molecules $\sigma=2\pi r^2$ ($r \equiv$ radius) and the number of particles per volume n . This determines the mean free path λ of a molecule without interacting with other molecules.

$$\lambda = \frac{1}{\sigma \cdot n} \approx \frac{1}{2\pi r^2 [cm^2] \cdot n [1/cm^3]}$$

The average distance $\bar{R}$ a molecule can travel after N collisions:

$$\bar{R} = \lambda \cdot \sqrt{N}$$

Because the time is proportional to the number of collisions:

$$\begin{aligned} t &\propto N \\ \Rightarrow \bar{R} &\propto \lambda \sqrt{t} \end{aligned}$$

$$t = \frac{\bar{R}^2}{6D}$$

$D \propto \lambda \sqrt{3mkT} \equiv$ diffusion constant of molecule through medium

$$D = \frac{kT}{6\pi r \eta}$$

r is the radius of the molecule, kT the thermal energy, and η is the viscosity coefficient of the surrounding medium.

The diffusion coefficient depend strongly on the viscosity of the surrounding medium. Tables show typical diffusion coefficients for molecules in air and water.

Diffusion Constants of Small Molecules in Air at Atmospheric Pressure

Molecule	Temp. (°C)	D (cm ² /sec)
Hydrogen	0°	.634
Water vapor	8°	.239
Oxygen	0°	.178
Carbon dioxide	0°	.139
Alcohol vapor	40°	.137

Diffusion Constants of Molecules in Water (20°C)

Molecule	M $\left(\frac{\text{gram}}{\text{mole}}\right)$	Radius (Å)	D (cm ² /sec)
Water (H ₂ O)	18	~ 1.5	2.0×10^{-5}
Oxygen (O ₂)	32	~ 2	1.0×10^{-5}
Urea CO(NH ₂) ₂	60	~ 4	1.12×10^{-5}
Glucose (C ₆ H ₁₂ O ₆)	180	~ 5	6.7×10^{-6}
Ribonuclease	13,683	~18.0	1.2×10^{-6}
β-lactoglobulin	35,000	~27.4	7.82×10^{-7}
Hemoglobin	68,000	~31.0	6.9×10^{-7}
Catalase	250,000	~52.2	4.1×10^{-7}
DNA	6,000,000		$.13 \times 10^{-7}$
Bushy stunt virus	10,700,000		1.15×10^{-7}
Tobacco Mosaic Virus	50,000,000		$.39 \times 10^{-7}$

This allows to calculate the time necessary for an oxygen molecule to diffuse through the entire aveoli bubble and through the aveoli membrane into the capillaric blood vessels.

transport time through bubble:

$$\bar{R} \approx 0.1 \text{ mm}, D \approx 0.18 \text{ cm}^2/\text{s}$$

The diffusion coefficient is approximated for diffusion in air at atmospheric pressure.

$$t_{alveoli} = \frac{10^{-4}[\text{cm}^2]}{6 \cdot 0.18[\text{cm}^2/\text{s}]} = 10^{-4}[\text{s}]$$

transport time through membrane:

$$\bar{R} \approx 0.005 \text{ mm}, D \approx 1 \cdot 10^{-5} \text{ cm}^2/\text{s}$$

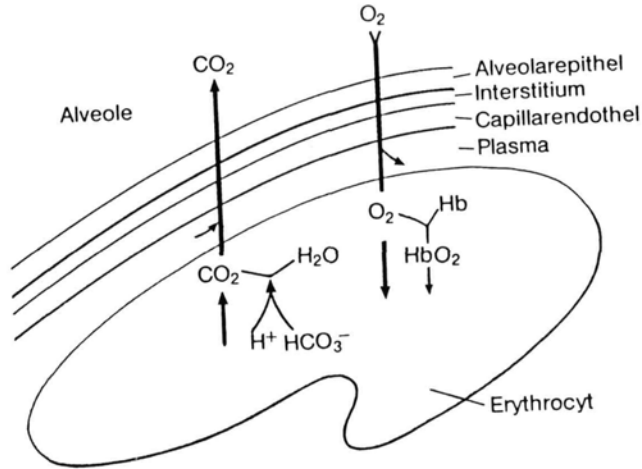
The diffusion coefficient is approximated for diffusion in water.

$$t_{membrane} = \frac{2.5 \cdot 10^{-7}[\text{cm}^2]}{6 \cdot 10^{-5}[\text{cm}^2/\text{s}]} = 4.2 \cdot 10^{-3}[\text{s}]$$

The average transit time for oxygen molecules through the aveoli bubble into the capillaric blood stream is about $t_{tr} \approx 4 \text{ ms}$ and is mainly determined by the diffusion through the membran layer between aveoli and blood vessels because of its smaller viscosity.

transit time of blood through the aveoli region

After its diffusion into the capillarie blood system the oxygen is chemically bound to hemoglobin in the blood. The transit time of the blood through the exchange region must be therefore sufficiently long to allow this process to happen.



Length of the capillary close to the aveola bubble: $L \approx 10^{-2} \text{cm}$

Average speed of blood through capillarie vessels: $v \approx 0.1 \text{cm/s}$

$$t_{\text{blood}} = \frac{L}{v} = \frac{10^{-2} [\text{cm}]}{0.1 [\text{cm/s}]} \approx 0.1 [\text{s}]$$

sufficient time for exchange process.

Diffusion by Partial Pressure Difference

The partial pressures of the O_2 , P_{O_2} and CO_2 , P_{CO_2} content in the lung are important for the gas exchange between aveoli and blood vessels. This can be derived from *Dalton's law* that the total pressure in a gas is equal to the sum of its partial pressures:

$$P = \sum_i P_i = P_{O_2} + P_{N_2} + P_{CO_2} + P_{H_2O}$$

and that the partial pressure is equal to the total pressure times the fraction of the volume $f_i = V_i/V_{tot}$

This yields for the gas pressure in the lung

$$P_{lung} = P_{O_2} + P_{N_2} + P_{CO_2}$$

$$P_{O_2} = f_{O_2} \cdot (P - P_{H_2O}) = \frac{V_{O_2}}{V_{tot}} \cdot (P - P_{H_2O})$$

P_{H_2O} is the vapor pressure of water

P is the outer atmospheric pressure

EXAMPLE At ground level the total pressure of the human lung is determined by $P_{N_2}=75.5$ [kPa], $P_{O_2}=14.0$ [kPa], $P_{CO_2}=5.3$ [kPa], and $P_{H_2O}=6.3$ [kPa]. The total atmospheric pressure at a height of 22 km is $P_{tot}=6.5$ [kPa]!

$\Rightarrow$ partial pressure of oxygen (and all other gases) in the lung is zero because the total pressure is determined by the vapor pressure of water only.

$\Rightarrow$ breathing is impossible at large heights!

To calculate the O_2 and CO_2 fractions in the aveoli, the balance between the inhaled, the absorbed, and the exhaled oxygen amount has to be determined. With an inhaled oxygen flow of

$$f_{iO_2} \cdot F_{tot} \quad (f_{iO_2} \equiv \text{oxygen fraction in inhaled air,}$$

$$F_{tot} \equiv \text{total gas ventilation rate}),$$

an exhaled amount of

$$f_{avO_2} \cdot F_{tot} \quad (f_{avO_2} \equiv \text{oxygen fraction in alveoli})$$

the rate of oxygen absorption by the aveoli, F_{O_2} is:

$$F_{O_2} = f_{iO_2} \cdot F_{tot} - f_{avO_2} \cdot F_{tot}$$

the rate of carbon dioxide extraction from the blood, F_{CO_2} is:

$$F_{CO_2} = f_{avCO_2} \cdot F_{tot}$$

this yields for the oxygen and carbon dioxide fractions in the aveoli:

$$f_{avO_2} = f_{iO_2} - \frac{F_{O_2}}{F_{tot}}$$

$$f_{avCO_2} = \frac{F_{CO_2}}{F_{tot}}$$

for typical values of $F_{O_2} \approx 0.28 \text{ l/min}$, $F_{CO_2} \approx 0.23 \text{ l/min}$, $F_{tot} \approx 4.1 \text{ l/min}$ and the atmospheric fraction on oxygen $f_{i_{O_2}} \approx 0.21$ standard values for the oxygen and carbon dioxide fractions in the aveoli can be obtained

$$f_{av_{O_2}} \approx 0.14 \qquad f_{av_{CO_2}} \approx 0.056$$

This results in partial pressures using Dalton's law:

$$P_{av_{O_2}} = P_{i_{O_2}} - \frac{F_{O_2}}{F_{tot}} \cdot (P - P_{H_2O})$$

$$P_{av_{CO_2}} = \frac{F_{CO_2}}{F_{tot}} \cdot (P - P_{H_2O})$$

with $P \approx 760 \text{ mm Hg}$, $P_{H_2O} \approx 47 \text{ mm Hg}$,
 $P_{I_{O_2}} \approx 150 \text{ mm Hg}$ ($0.2 \cdot P$),

$$P_{av_{O_2}} = 150[\text{mmHg}] - \frac{0.28[\text{l/min}]}{4.1[\text{l/min}]} \cdot (760[\text{mmHg}] - 47[\text{mmHg}]) = 100[\text{mmHg}]$$

$$P_{av_{CO_2}} = \frac{0.23[\text{l/min}]}{4.1[\text{l/min}]} \cdot 713[\text{mmHg}] = 40[\text{mmHg}]$$

The large difference in oxygen and carbon dioxide partial pressure results mainly from the ventilation rate F_{tot} for the aveoli.

Diffusion into the Blood Stream

Diffusion processes due to differences in the concentration level are described by Fick's first diffusion law:

$$\frac{dn}{dt} = D \frac{A}{d} \cdot \Delta C$$

n is the number of diffusing molecules; D is the diffusion coefficient; A , d is the area and thickness of membrane; ΔC is the concentration difference.

Diffusion is proportional to concentration gradient

Diffusion law can be reformulated in terms of the difference in partial pressures ΔP

$$\frac{dn}{dt} = K \frac{A}{d} \cdot \Delta P$$

K is the Krogh diffusion coefficient, which is proportional to D but has different values and dimensions.

Diffusion is proportional to the gradient in partial pressure

For the gas mixture in the aveoli

$$K_{CO_2} \approx 23 K_{O_2};$$

the diffusion coefficient for carbon dioxide the diffusion coefficient is 23-times larger than for oxygen (due to the difference in partial pressures!)

The equilization between the O₂ partial pressure in the aveoli and in the blood capillar follows an exponential behavior (this results from solving the Fick differential equation for the diffusion.

$$\frac{d\Delta P}{dt} = -\frac{1}{\tau_0} \cdot \Delta P(t)$$

$$\Delta P(t) = \Delta P(t_0) \cdot e^{-\frac{t}{\tau_0}}$$

τ_0 is the time constant for the diffusion process which depends on the diffusion coefficient and the volume of the aveoli bubble V_1 and the volume of the capillar blood vessel V_2 .

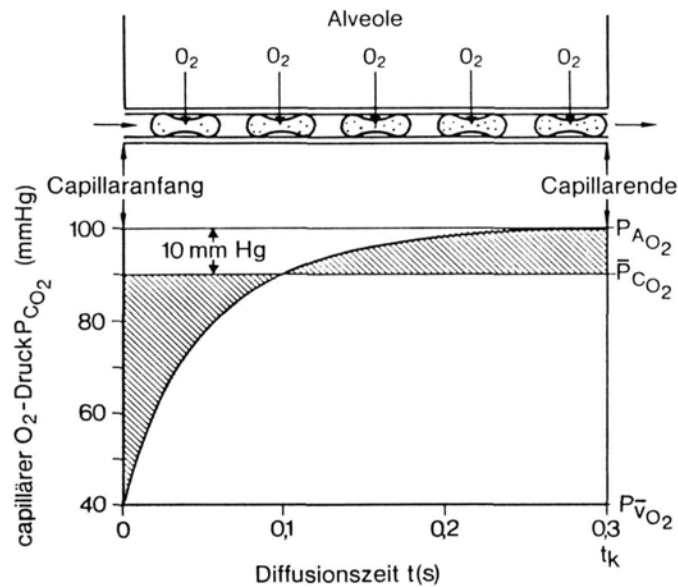
$$\tau_0 = -\frac{1}{D_L} \cdot \frac{V_1 V_2}{V_1 + V_2}$$

With the assumption that all of the O_2 flow into the aveoli is absorbed through the membranes:

$$\frac{dn_{O_2}}{dt} = F_{O_2} = K \frac{A}{d} \cdot \Delta P(t) = D_L \cdot \Delta P(t)$$

$D_L = K \cdot \frac{A}{d}$ is called the O_2 diffusion coefficient of the lung.

$\Delta P(t)$ is the O_2 partial pressure difference between the aveoli volume and the capillarie blood stream which changes with time over the diffusion process until the O_2 partial pressures (or O_2 concentrations) in the aveoli and the blood vessel are equalized.



EXAMPLE: O₂ DIFFUSION COEFFICIENT OF AN AVERAGE LUNG

For simplification we may neglect the time dependence of the pressure equilization and adopt an average pressure difference $\bar{\Delta P}$ between the aveoli bubble and the blood vessel. This modifies the equation for the diffusion flow:

$$\frac{dn_{O_2}}{dt} = F_{O_2} = D_L \cdot \bar{\Delta P}$$

$\bar{\Delta P}$ is approximated as the mean value of the partial pressure difference averaged over the entire time of the diffusion process (see figure).

For the average adult the mean value of the partial pressure difference for oxygen is:

$$\bar{\Delta P} \approx 10 \text{ [mmHg]},$$

the oxygen flow for an adult at rest is:

$$F_{O_2} \approx 300 \text{ [ml/min]}.$$

using typical aveoli parameters $A=80 \text{ cm}^2$, $d=0.001 \text{ mm}$, the value for the O₂ diffusion coefficient is estimated:

$$D_L \approx 30 \text{ [ml/(min} \cdot \text{mmHg)]} \quad K = D_L \cdot (d/A) \approx 5 \cdot 10^{-6} \text{ [cm}^2\text{/s} \cdot \text{kPa]}.$$