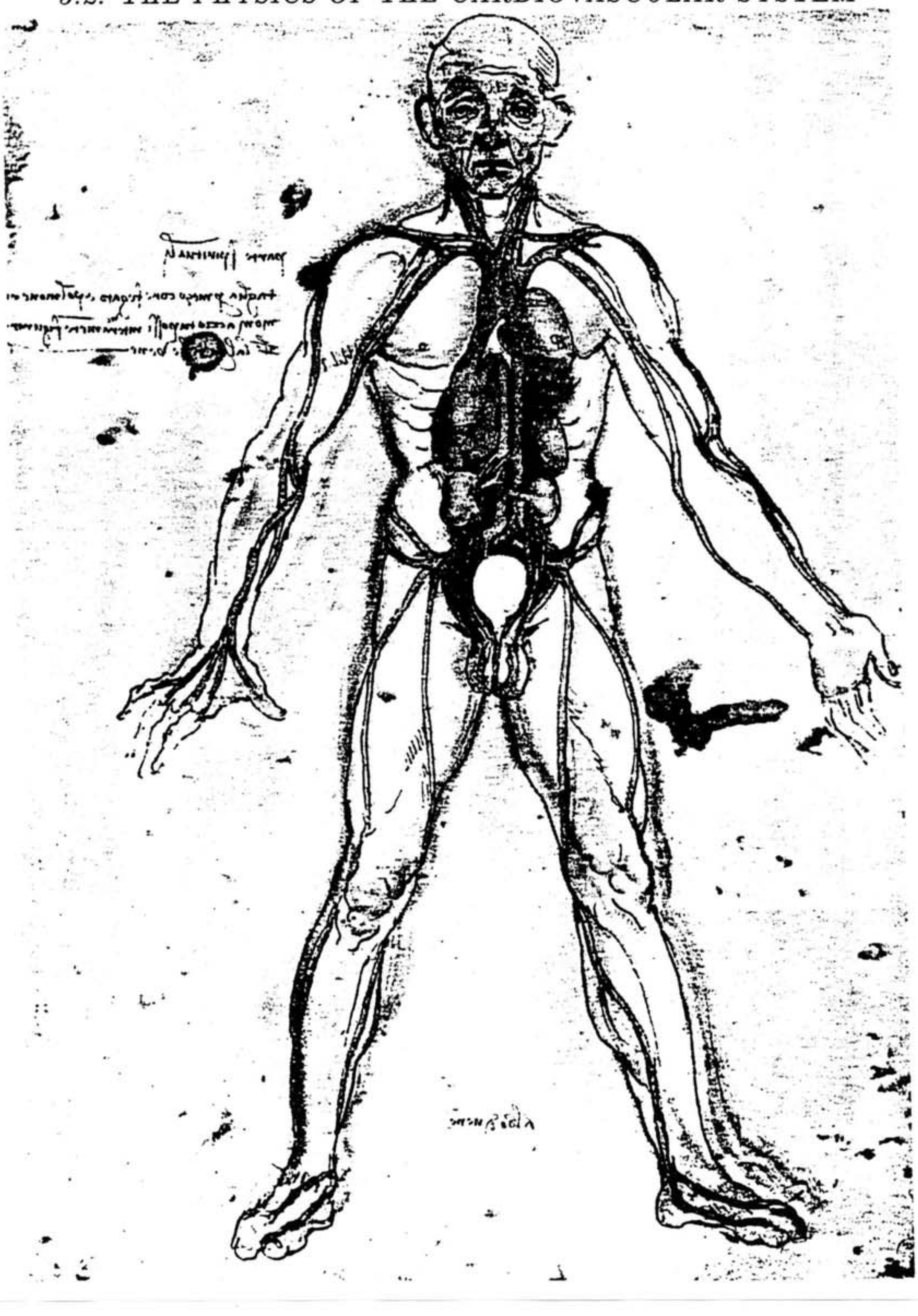
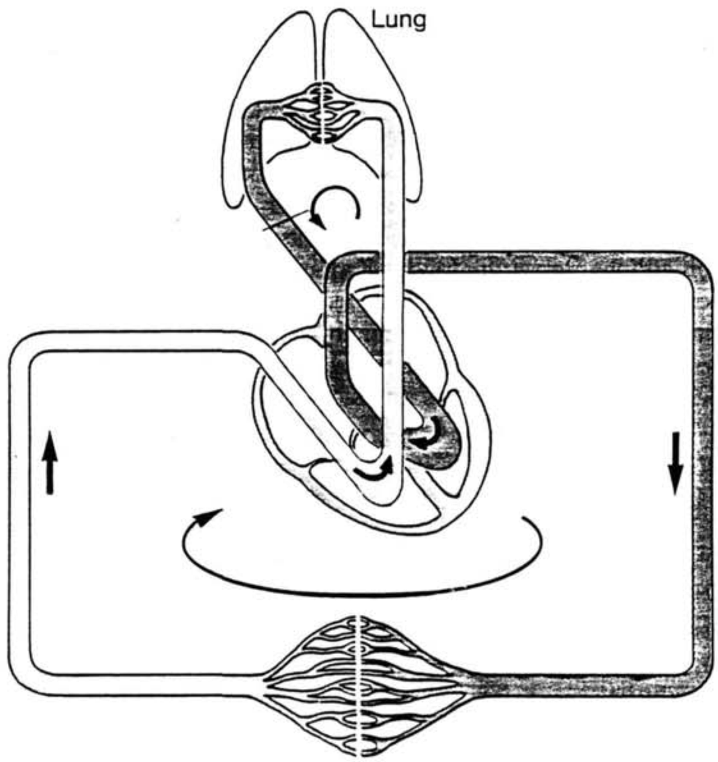
3. THE PRESSURE SYSTEM OF THE BODY



The blood system is the main transport system for fuel supply and disposal of body by-products.

It serves for distribution of food contents (from the digestive system) and of oxygen (from the respiratory system) to the body cells and it serves also as transport system for disposing the by-products (like CO_2) of the combustion process.



General Physiological Information about Blood

The total blood mass is ≈7% of the total body mass.
For an average person of a mass; m = 65 kg,

$$\Rightarrow \quad m_{blood} \approx 4.5 \text{ kg}.$$

blood density: $\rho_{blood} = 1.04 \cdot 10^3$ kg/m^3

$$\Rightarrow \quad V_{blood} \approx 4.4 \; l$$

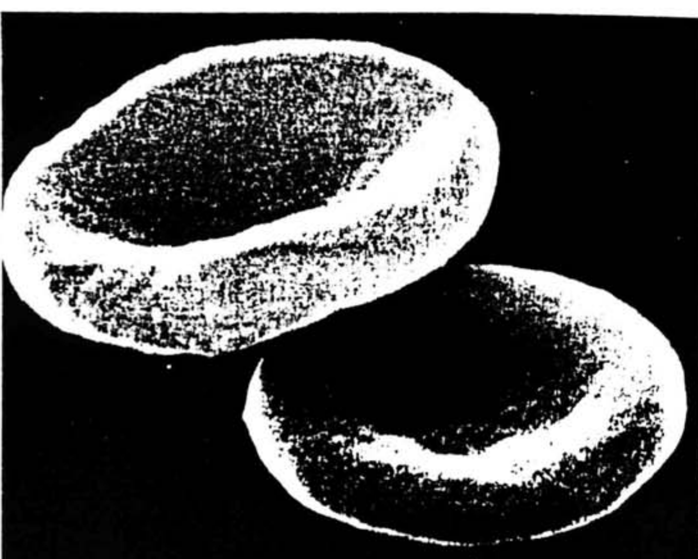
Main components of the blood are:

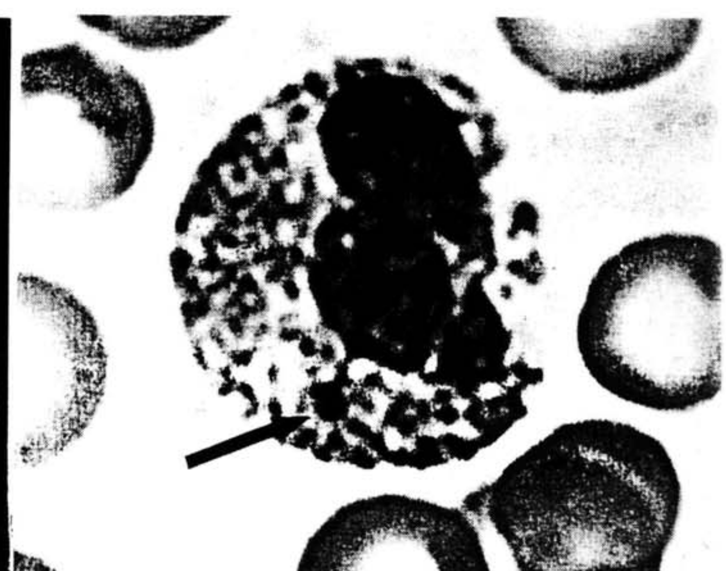
blood plasma, whitish liquid: 55% of volume,
$\approx$ 900 g H_2O/l, $\approx$ 70 g protein/l

red blood cells (erythrocytes),
$\approx 5{\cdot}10^6/\mu l^3$: 44% of volume,
disk shaped with 7 μm diameter, $\approx$ 2 μm thickness
determines the fluid characteristics of blood!

white blood cells (leukocytes), $\approx 7\ 10^3/\mu l^3$:
spherically shaped with $\approx$10-20 μm diameter
important for the human immune system,
increase of number of leucocytes in case of infections

platelets (thrombocytes), $\approx 3\ 10^5/\mu l^3$:
flat shaped with 4$\approx$ 2.5 μm length,$\approx$ 0.6 μm thickness.
important for blood clotting

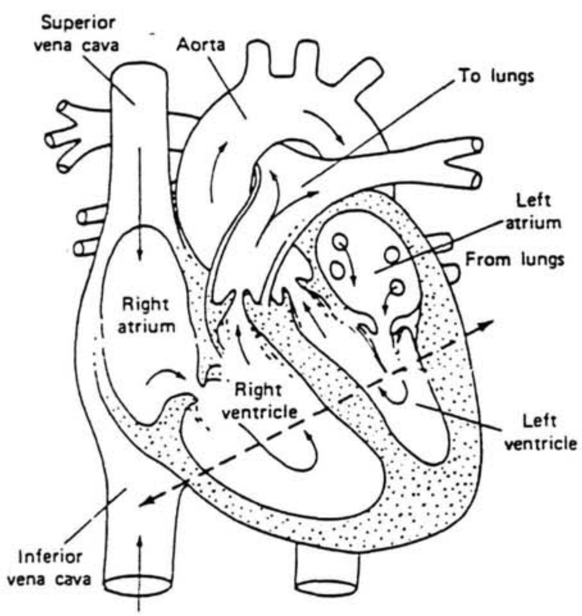




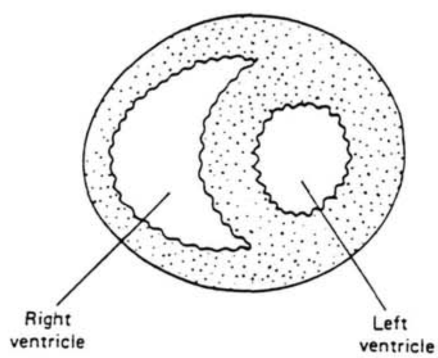
The Heart as Main Component of the Cardiovascular System

Blood vessels which transport blood away from the heart are called
arteries

Blood vessels which transport blood towards the heart are called
veins



Heart as double pump system:
left ventricle chamber is compressed by heart muscle, 17kPa≈125mm Hg blood is pumped through the aorta into the large systemic cycle (supply of all body functions) cycle closes at right atrium blood pumped by weak contraction, 0.8kPa≈6mm Hg to right ventricle. right ventricle chamber compresses with 3.5kPa≈25mm Hg, pumps blood through pulmonary arteries into the small pulmonary cycle (refreshing of O_2, disposing of CO_2) cycle closes at left atrium blood pumped by weak contraction (1kPa≈8mm Hg) to left ventricle.

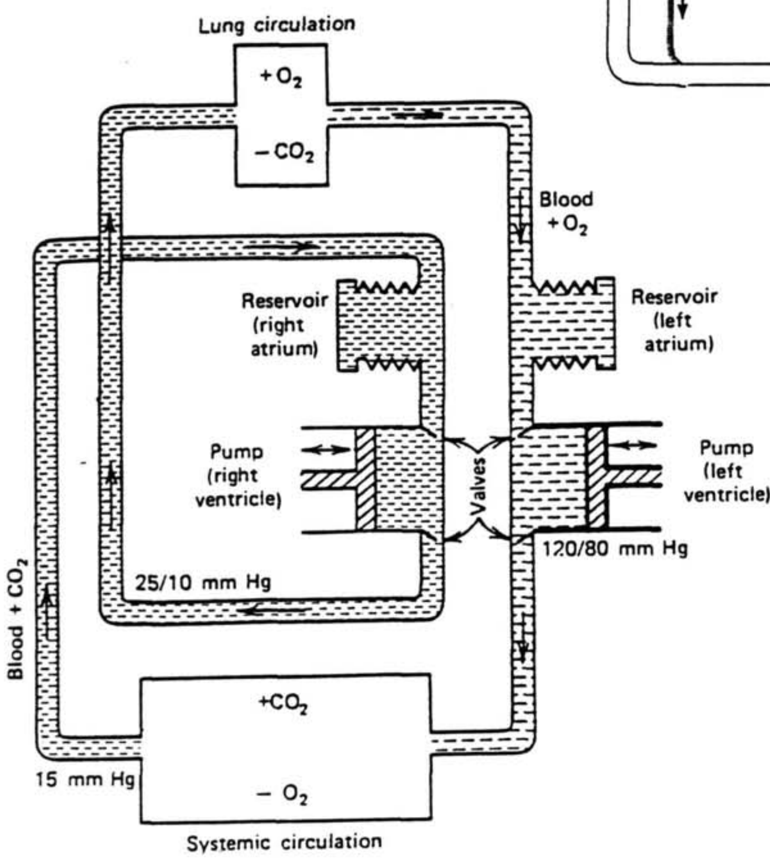


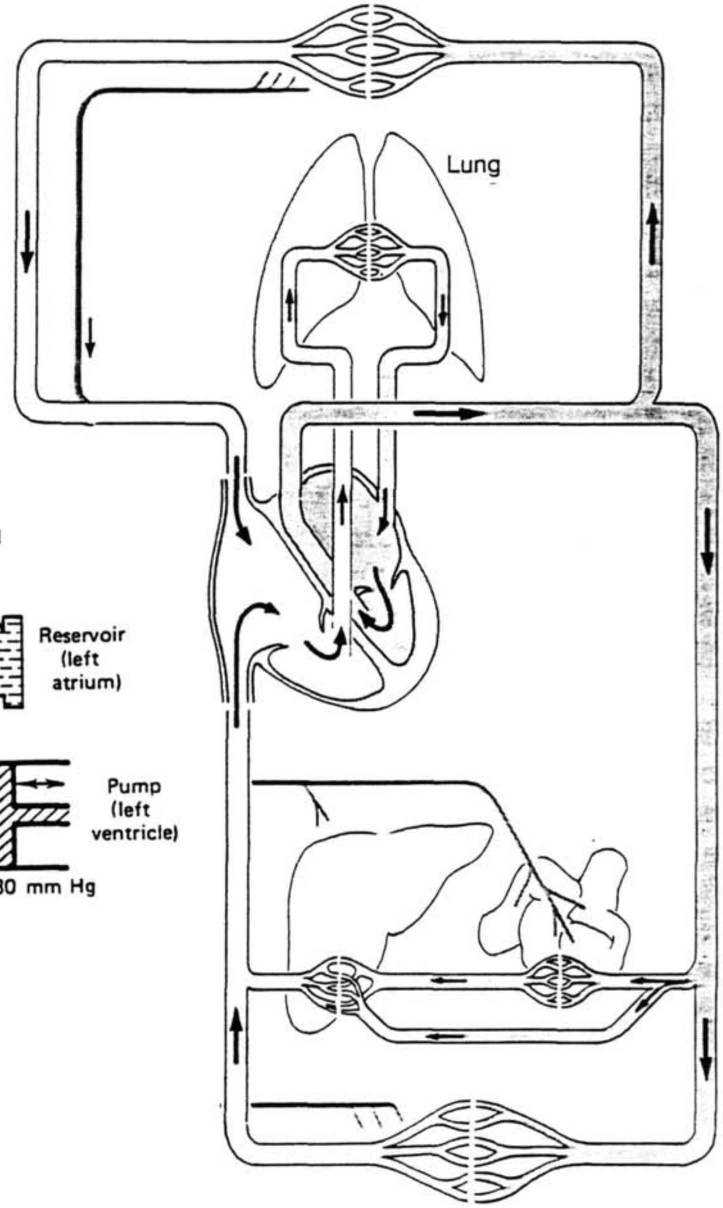
each contraction pumps ≈75 - 80 ml of blood

⇒ fraction of: t≈ $\frac{0.08l}{4.4l}$ ≈0.018/contraction (heart beat)

With an average heart beat frequence of 1 Hz:

⇒ timescale of ≈ 1 min/circulation.

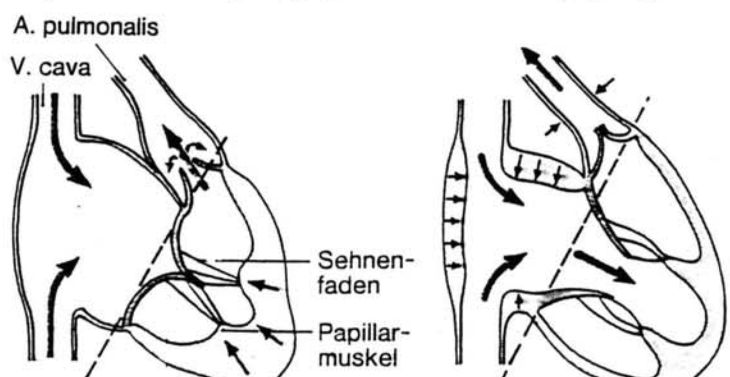




Because of the difference in volume between the two blood cycles, blood is not equally distributed: ≈84% in systemic cycle; ≈10% in pulmonaly cycle.

region	Volume [ml]	Volume [%]
heart	360	7.2
systemic cycle	4200	84
aorta and arteries	300	6.0
arterioles	400	8.0
capillares	300	6.0
small veins	2300	46
large veins	900	18
pulmonaly cycle	440	8.8
arteries	130	2.6
capillares	110	2.2
veins	200	4.0

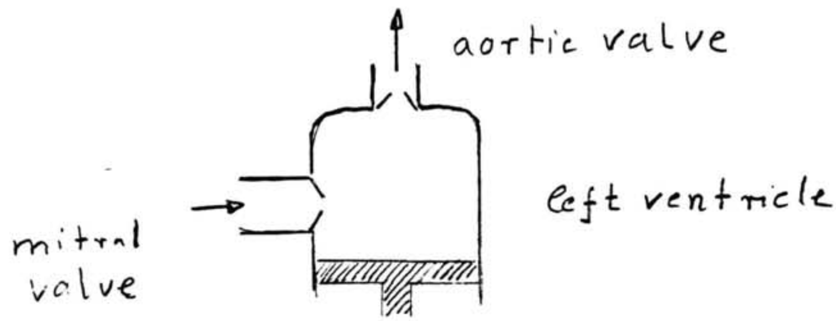
The flow direction of the blood is controlled by valves at the entrance (mitral valve (left), tricuspidal valve (right)) and exit of ventricles (aorta valve (left), pulmonal valve (right))



The valves are controlled by the pressure conditions in the ventricles caused by the contraction of the heart muscle.

Work of the Heart

The heart can be described by simple model of a pump.
The heart muscle contracts which causes a decrease in volume in the respective heart chamber
Increse in pressure closes inlet valve and opens outlet valve.



The amount of work for the heart muscle can be calculated from the force F of the muscle:

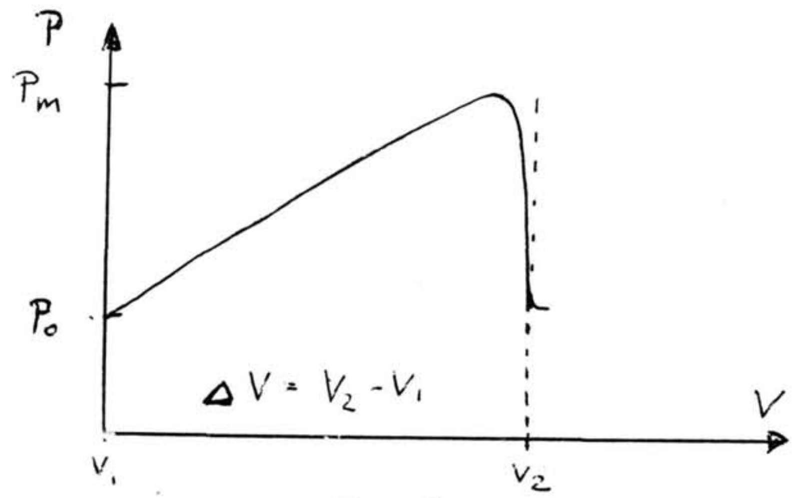
$$\Delta W = \int F dr$$

the work can be expressed in terms of pressure P on the surface area of the heart chamber.

$$\Delta W = \int \frac{F}{A} A dr = \int \frac{F}{A} dV = \int_{V_1}^{V_2} P dV$$

$V_2 - V_1 = \Delta V$ is the volume of the pumped blood during each compression.

Pressure increases from diastolic pressure P_0 slowly up to systolic pressure P_m and falls rapidly back to diastolic pressure, after the heart muscle releases.



$$P(V) = P_0 + \frac{P_m - P_0}{\Delta V} \cdot (V - V_1)$$

This equation describes the pressure-volume relation in the pump; to calculate the work it has to be integrated over the volume:

$$\Delta W = \int_{V_1}^{V_2} (P_0 + \frac{P_m - P_0}{\Delta V} \cdot (V - V_1)) dV$$

$$\Delta W = P_0 \cdot \Delta V + \frac{P_m - P_0}{2} \cdot \Delta V$$

$P_0 + 1/2 \cdot (P_m - P_0) \equiv \bar{P}$ represents the average blood pressure in the ventricles

For the present example $P_0 \approx$80 mm Hg, and $P_m \approx$120 mm Hg; this yields an average blood pressure of $\bar{P} \approx$100 mg Hg.

For a volume of $\approx$75 ml per contraction pumped by the heart the amount of work with each heart beat is:

$$\Delta W = 100[mmHg] \cdot 75[\overset{m}{\cancel{\mu}}l] = 1.33 \cdot 10^4[Pa] \cdot 7.5 \cdot 10^{-5}[m^3] = 1[J]$$

The power or work rate depends on the frequency of the heart beat; for an approximate average frequency of 1 Hz (one beat per second):

$$\frac{\Delta W}{\Delta t} = 13[kPa]\cdot 7.5\cdot 10^{-5}[m^3]\cdot 1[s^{-}1] = 1[W] = 0.24[cal/s] = 21[kcal/day]$$

this is the average power required for the heart beat, however, pressure is pulsed, peak power is about three times the average power!

efficiency of the heart as machine $\approx$ 12 - 20 %,
average value: $\bar{\epsilon} \approx 15$ %

$$\bar{\epsilon} \cdot \frac{\Delta U}{\Delta t} = \frac{\Delta W}{\Delta t}$$

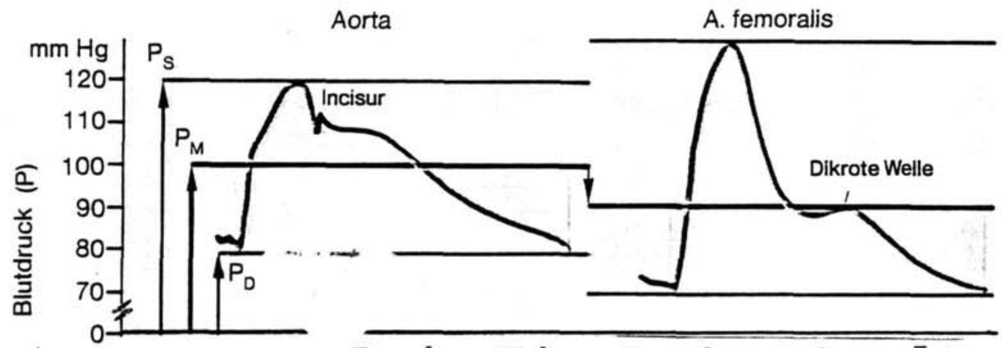
$$\frac{\Delta U}{\Delta t} \approx \frac{1}{0.015} \cdot 21[kcal/day] = 140kcal/day$$

in comparison with the basal metabolic rate:
$\Delta Q_{bas} \approx 70\ [kcal/hr] = 1680\ [kcal/day]$

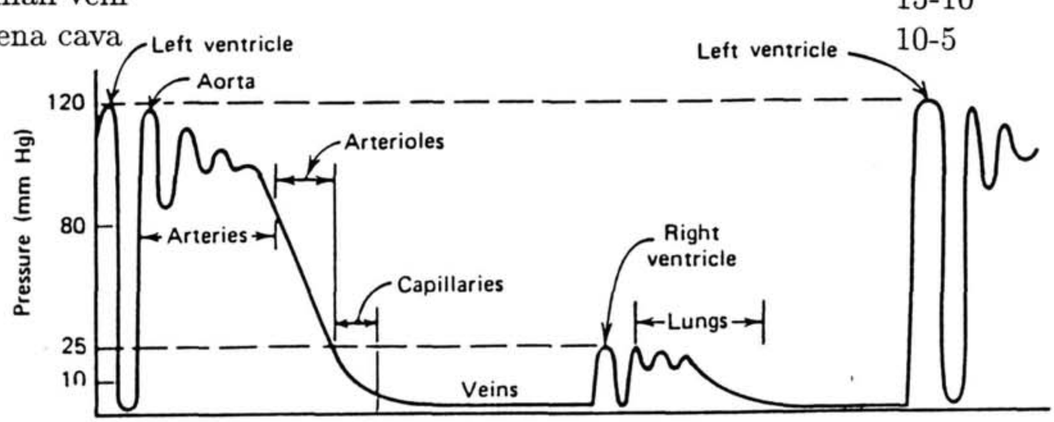
$\approx$ 9 % of the basal metabolic rate is due to the permanent heart activity. (fraction of body mass $\approx$0.5 %)

Pressure conditions in the cardiovascular system

There are drastic differences in pressure over the entire blood vessel system.
Peak pressure conditions are reached in the left ventricle during contraction of the heart muscle,
⇒ Pressure pulse travels towards aorta and distributes pressure to arterial system, pressure drops with increasing distance to heart.



region	P_{syst} $[mmHg]$	P_{diast} $[mmHg]$	$\bar{P}$ $[mmHg]$
right atrium			5
right ventricle	25	5	
Artery pulmonalis	25	10	
left atrium			10
left ventricle	120	10	
aorta	120	70	100
artery			95-90
arterioles			70-35
small capillary			30-15
small vein			15-10
vena cava			10-5



pressure conditions in the circulatory system.

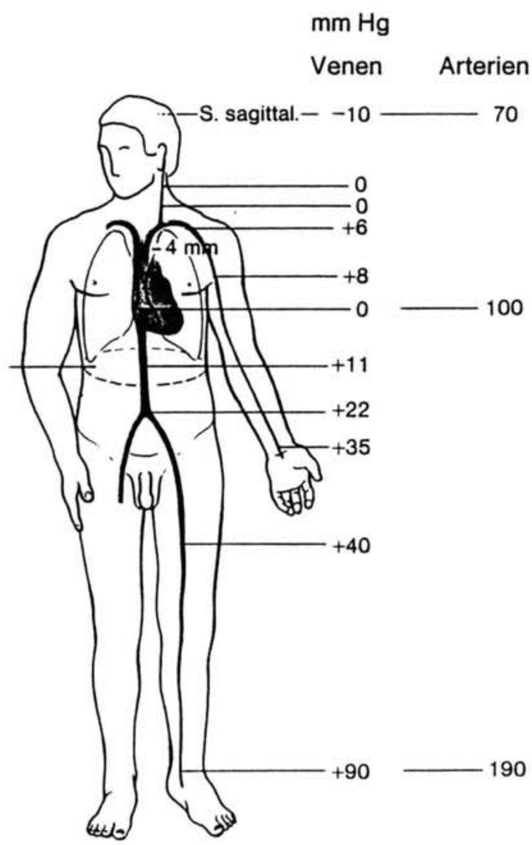
Influence of gravitation

gravitational forces influence the pressure conditions in the circulatory blood system.

$$P_g = h \cdot \rho \cdot g; \quad \rho = 1.04 \cdot 10^3 kg/m^3;$$

Additional pressure at the feet due to the height of the column $h_{(feet-heart)} \approx 125$ cm:

$$P_g = 1.25[m] \cdot 1.04 \cdot 10^3[kg/m^3] \cdot 9.81[m/s^2] = 12.5\ [kPa] \approx 90\ [mmHg]$$



at the arterial side the mean value of the arterial pressure $\bar{P_{art}} \approx 100$ mm Hg has to be added.

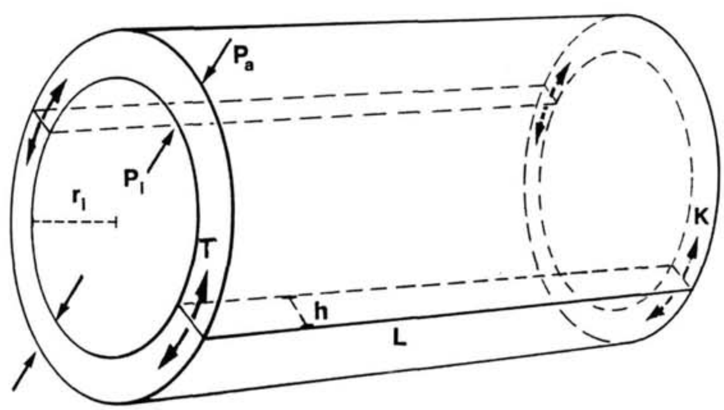
EXAMPLE THE BLOOD PRESSURE IN THE BRAIN FOR A PERSON STANDING ON HIS HEAD

$$P_g \approx 0.5[m] \cdot 1.04 \cdot 10^3[kg/m^3] \cdot 9.81[m/s^2] = 5\ [kPa] \approx 38\ [mmHg]$$

Transmural pressure across blood vessel walls

Blood vessels have elastic walls which have the tendency to contract
$\Rightarrow$ vessels may collaps due to wall tension

pressure pulse travels along blood vessel
$\Rightarrow$ vessels may burst due to rapid pressure increase



transmural pressure: $P_t = P_a - P_i$
increase in transmural pressure widens the diameter but elasticity prevents burst.

transmural pressure causes tangential tension T_h which can be described by the *Laplace law* in terms of the inner radius r_i and the wall thickness h:

$$T_h = P_t \cdot \frac{r_i}{h}[N/m^2]$$

tension is proportional to pressure and inner radius, but inverse proportional to the wall thickness.

pressure conditions in the various blood vessels of the cardiovascular system;

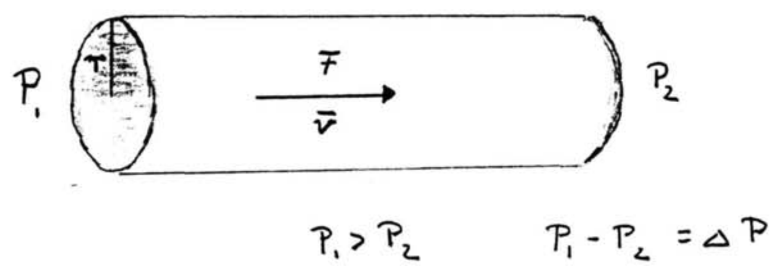
region	r_i [mm]	r/h	P_t [kPa]	T_h [kPa]
aorta	12	8	13.3	106
artery	0.5-3	3-7	11	33-77
arterioles	0.01-0.1	1-5	7	7-35
capillares	0.003	5-8	3.3	17-26
small veins	0.01-0.25	7-10	1.6	11-16
veins	0.75-7.5	7-10	1.3	9-13
vena cava	17	10-15	1	10-15

large radii are correlated with thicker walls to balance the tensions.

The Blood Flow Conditions in the Cardiovascular System

the pressure difference ΔP between two points along a tube originates a flow F:

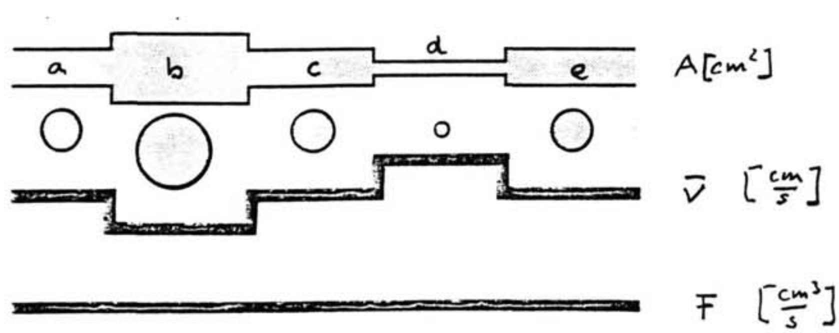
$$F \propto \Delta P$$



the flow is determined by the average velocity $\bar{v}$ of the particles and by the cross sectional area of the tube $A{=}\pi \cdot r^2$

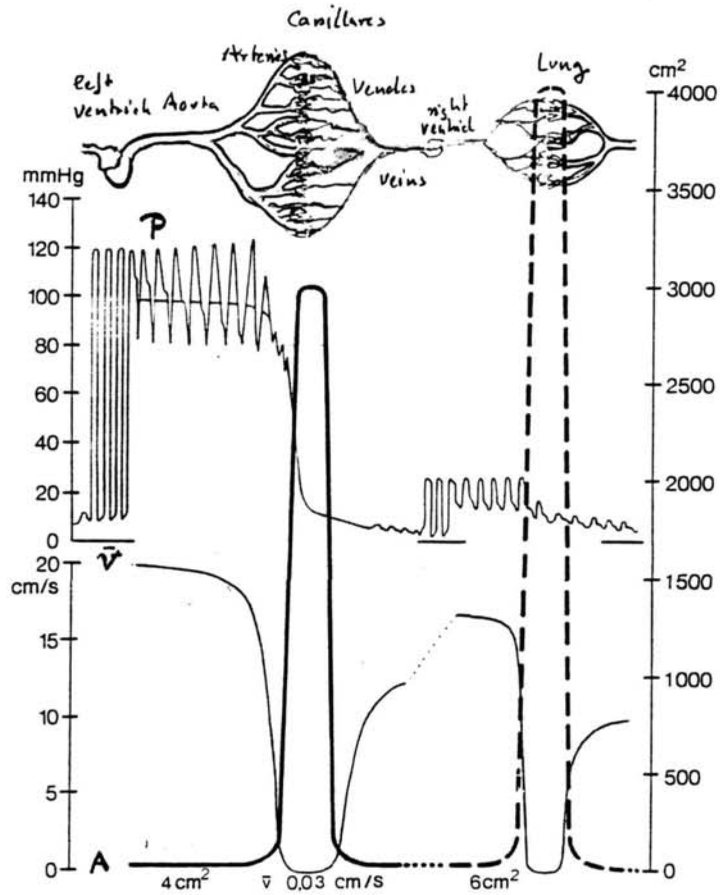
$$F = \bar{v} \cdot A \left[\frac{l}{t}\right]$$

The continuity law for a flow states that independent of the tube radius the flow is constant.



$$F = A_a \cdot \bar{v}_a = A_b \cdot \bar{v}_b = A_c \cdot \bar{v}_c = ... = const$$

This means that the average velocity of the particles changes inverse proportional with the square of the tube radius.



The maximum average flow velocity is $\bar{v_A} \approx$ 20-30 cm/s in the aorta and main arteries. In the capillar mesh the average velocity drops down to $\bar{v_C} \approx$0.03 cm/s due to the decrease in radius.

For an average length of a capillar vessel of L=750 μm = 0.75 mm the time required for a red blood cell to pass the cappilar is;

$$t = \frac{L}{\bar{v_A}} = 2.5s$$

The resistance R of the flow F is determined by the pressure difference ΔP between two points in a tube:

$$R = \frac{\Delta P}{F} \left[\frac{mmHg}{l/s}\right]$$

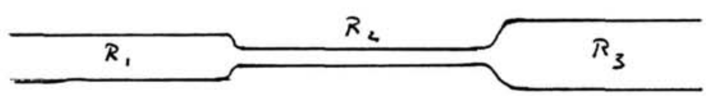
Determination of the resistance requires a knowledge of the flow F.

EXAMPLE The total Resistance of the Circulatory Blood System

As the blood flows from the aorta through the major arteries, the small arteries, the capillares, and the veins to the right atrium the pressure drops from 100 mm Hg to nearly zero. If the flow rate is approximately $F \approx 95$ ml/s the total resistance is:

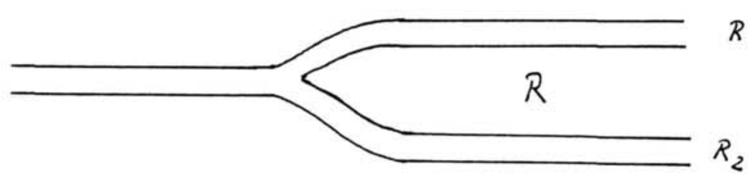
$$R_{tot} = \frac{\Delta P}{F} = \frac{100[mmHg]}{95[ml/s]} = \frac{1.33 \cdot 10^4[Pa]}{0.095[l/s]} = 1.4 \cdot 10^5[Pa \cdot s/l]$$

The resistance of a system of tubes can be calculated using the following relations:



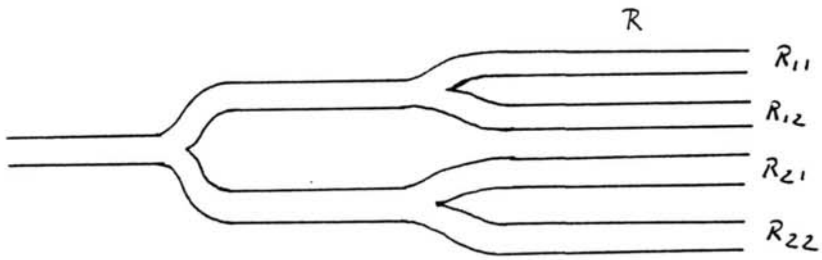
tubes in series

$$R = R_1 + R_2 + R_3$$



tubes parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}; \rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



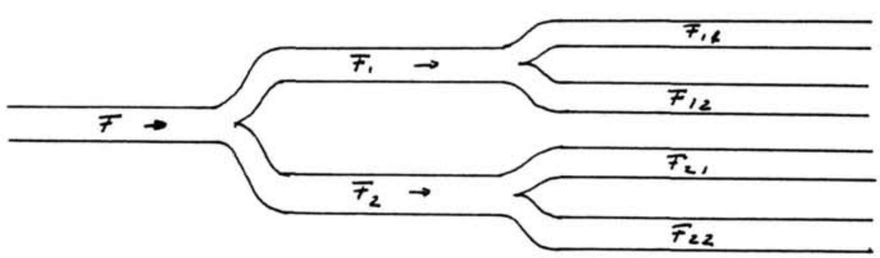
tubes prallel

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{R_{11}}+\frac{1}{R_{12}}+\frac{1}{R_{21}}+\frac{1}{R_{22}}$$

By using the expression for the flow in the case of a branching we obtain the relation between the main flow F and the branch flows F_1 and F_2 and the flows in the further branches F_{11}, F_{12}, F_{21}, and F_{22}:

$$F=\frac{\Delta P}{R}=\frac{\Delta P}{R_1}+\frac{\Delta P}{R_2}=F_1+F_2=F_{11}+F_{12}+F_{21}+F_{22}$$

this indicates that in a branching of the tube system the flow branches F_1, F_2 are inverse proportional to the resistences R_1 and R_2.



The flow F of a fluid is determined by the viscosity of the liquid η [Pa·s]

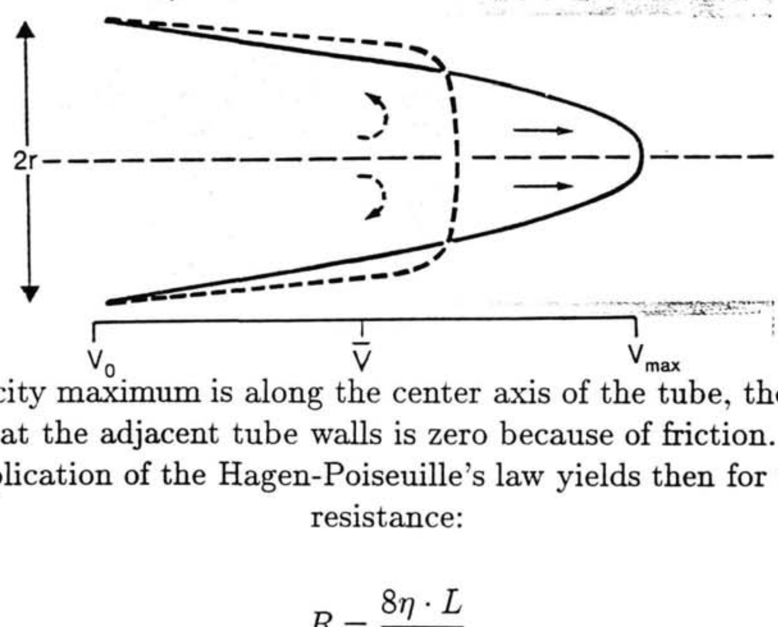
The viscosity is a temperature dependent material constant (different for each liquid) which reflects the friction between the fluid components.

The international standard unit for the viscosity is:
1 poise = 1 Pa·s

For a **laminar flow** (flow without turbulences, particles move all in one direction) the flow through a cylindrical tube with radius r and length L is described by the **Hagen-Poiseuille's law**:

$$F = \frac{\pi \cdot r^4}{8\eta \cdot L} \cdot \Delta P$$

The velocity profile for a viscous or laminar flow is shown in the figure:



The velocity maximum is along the center axis of the tube, the velocity at the adjacent tube walls is zero because of friction.
The application of the Hagen-Poiseuille's law yields then for the flow resistance:

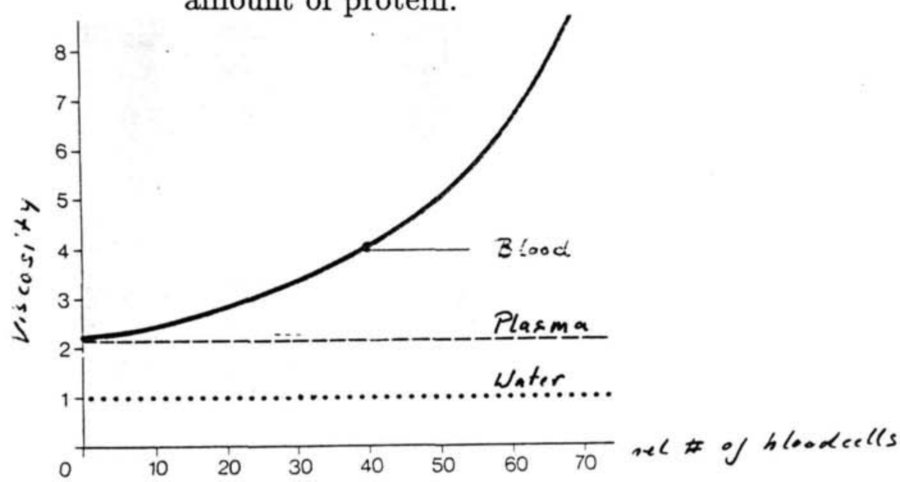
$$R = \frac{8\eta \cdot L}{\pi \cdot r^4}$$

The resistance is directly proportional to the viscosity of the fluid.

The following table gives the viscosity of various fluids at a temperature of 20°C (blood at 37°C)

FLUID	η	[mPa·s]
water	1.0	
glycerin	1.4	
engine oil	200	
blood	4.0	

Because blood is a mixture of fluids (water, protein) and particles (red blood cells, white blood cells etc) its viscosity is not an absolute constant but varies strongly with the number of blood cells and the amount of protein.



The resistance for the blood vessels is usually calculated for the viscosity of 'normal' blood, η=4 mPa·s.

For cold temperature conditions the viscosity increases.

$$\eta(0^oC) \approx 2.5 \cdot \eta(37^0C)$$

this increases the resistence in the blood flow. For example, less blood flow through cold feet and hands.

EXAMPLE The Resistance of the Aorta

$$L_{aorta} \approx 30 \text{ cm}, \; r_{aorta} \approx 1.3 \text{ cm}$$

$$R = \frac{8\eta \cdot L}{\pi \cdot r^4} = \frac{8 \cdot 4 \cdot 10^{-3} \cdot 0.30[Pa \cdot s \cdot m]}{\pi \cdot (0.013)^4[m^4]} = 1.07 \cdot 10^5[Pa \cdot s/m^3]$$

$$\Rightarrow \quad 107 \; [Pa \cdot s/l]$$

only a small fraction of the total resistance in the circulatory system (see previous example) because of the large diameter of the aorta.

To calculate the resistance of a vascular bed of N blood vessels we make the following assumptions:

- all arteries have the same diameter
- all arteries in the vascular bed are parallel
- all capillares are parallel
- all veins are parallel

if one vessel has the resistance R_i, the total resistance of the vascular bed is:

$$R_{tot} = \frac{R_i}{N}$$

The total flow F distributes over the N vessels: $F = N \cdot F_i$

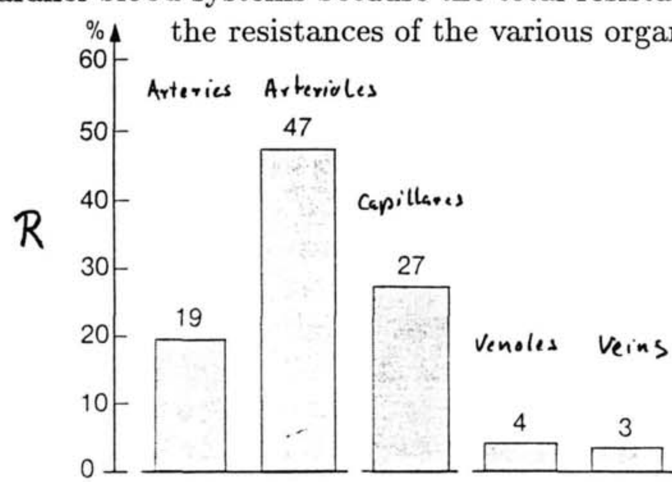
EXAMPLE The Resistance of a Capillary System

In the capillary system of the lung the pressure gradient is:
$\Delta P \approx$ 8mm Hg using a flow of $F \approx 75\ ml/s$
the resistance $R_{lung} \approx 1.1 \cdot 10^4$ Pa$\cdot s/l$.
Taking the dimensions of a single capillaric vessel to be $\approx$ 1 cm long with an average diameter of 20 μm the resistance of a single capillar is:
$R_c \approx 6.4 \cdot 10^{11}$ [Pa$\cdot s/l$]
This requires a capillaric net of $N \approx 6 \cdot 10^7$ capillar vessels. (Mainly caused by the small average diameter).

The table lists the average resistances for the various body systems

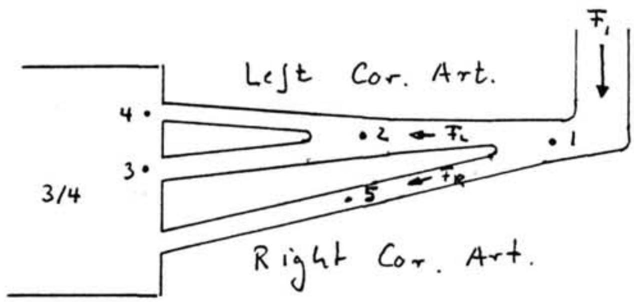
organ	F $[ml/s]$	R $[Pa \cdot s/ml]$
brain	13	1025
coronary vessels	14	3330
muscles	20	670
skin	8	1670
organs	10	1330
systemic cycle	95	140
pulmonal cycle	96	11

Comparing the resistances in the capillaric system of an organ with the total resistance indicates already the distribution of the flow into numereous blood vessels, it also indicates that most of the organs are on parallel blood systems because the total resistance is smaller than the resistances of the various organs.



EXAMPLE THE BLOODFLOW IN THE CORONARY SYSTEM

Consider the coronary artery and vein portion of the human circulatory as shown in the figure.



The flow resistance of 1 cm length of each of the left and right coronary arteries is
R_{12}=R_{15}=4 mm Hg s/cm^3.
The total flow at point 1 is F_1=6 cm^3/s. The radius of each coronary is r=0.1 cm. At point 2 the left coronary artery splits into two arteries of each r_i=0.07 cm with a flow resistance of:
R_{23}=R_{24}=16 mm Hg s/cm^3 each.
This allows to calculate the total flow resistance $R_{1\to3/4}$ between point 1 and 3/4 through the entire artery system.

$$R_{2\to3/4} = \frac{16\cdot16}{32} = 8mmHg\cdot s/cm^3$$

$$R_{1\to2\to3/4} = 8+4 = 12mmHg\cdot s/cm^3$$

$$R_{1\to3/4} = \frac{12\cdot8}{20} = 4.8mmHg\cdot s/cm^3$$

This allows to determine the flow rates through the left and right coronary arteries (at points 2,5) and the flow rate between point 2 and 3/4 through each section of the left artery.

$$F_L + F_R = F_1$$

The pressure gradient between point 1 and 3/4 is:

$$P_1 - P_{3/4} = F_1 \cdot R_{1\rightarrow 3/4} = 6 \cdot 4.8mmHg = 28.8mmHg$$

The flow through the left artery system :

$$F_{1\rightarrow 2\rightarrow 3/4} = \frac{28.8mmHg}{12mmHg \cdot s/cm^3} = 2.4cm^3/s$$

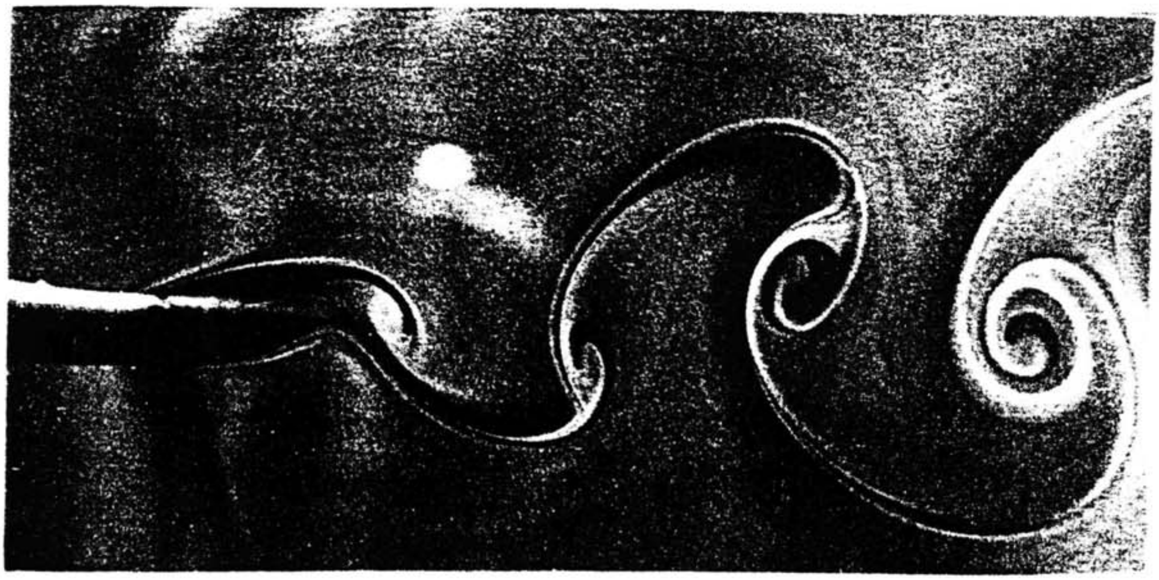
The flow through the righ artery is:

$$F_{1\rightarrow 5\rightarrow 3/4} = \frac{28.8mmHg}{8mmHg \cdot s/cm^3} = 3.6cm^3/s$$

The flow through the righ artery system is larger because there is less splitting of the blood vessels.

Turbulent Flow

If the velocity of a fluid increases above a critical velocity the laminar flow becomes turbulent. For this case the linear relation between pressure gradient and flow becomes invalid because additional local pressure differences are caused by the turbulences.



The critical velocity for the turbulent flow in a tube depends on the viscosity η, the density ρ and the average velocity $\bar{v}$ of the fluid as well as on the radius r of the tube.
The flow condition of a fluid is characterized by Reynold's number N_R, which is defined by:

$$N_R = \frac{2 \cdot r\rho \cdot \bar{v}}{\eta}$$

The Reynold's number is dimensionless.

The flow will be laminar if the Reynold's number is less than 2000, it will be turbulent if the Reynold's number will be greater than 3000. Between these numbers the flow conditions are unstable and may alter.

EXAMPLE TURBULENT FLOW IN THE BLOOD VESSELS

The critical value for the Reynold's number N_R is reached in the aorta during the opening of the aortic valve, when momentarily a peak velocity of 50 cm/s (in extreme cases up to 100 cm/s) can be reached.

$$N_R = \frac{2 \cdot r\rho \cdot \bar{v}}{\eta} = \frac{2(0.01[m])(1060[kg/m^3])(0.5[m/s])}{4 \cdot 10^{-3}[Pa \cdot s]} = 2650$$

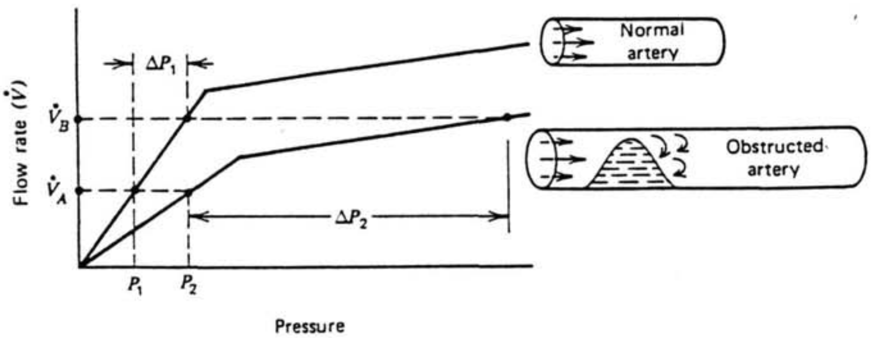
Critical conditions for turbulences are reached!
(**origin for heart sound heard with stethoscope**)
For the average velocity of 30 cm/s the Reynold's number is N_R=1590 and the flow is laminar.

Because of the relation between the average flow velocity $\bar{v}$ and the flow rate F:

$$\bar{v} = \frac{F}{A} = \frac{F}{\pi \cdot r^2} = \frac{r^2}{8 \cdot \eta \cdot L} \cdot \Delta P$$

a reduction in the radius of the blood vessel will cause an increase in the average flow velocity.
A reduction of the aorta radius by 20% from r=1cm to r=0.8 cm increases the average flow velocity from
$\bar{v} = 30\ cm/s$ up to $47\ cm/s$ (calculated for a flow of F=95 ml/s).
This changes the Reynold's number from 1590 to N_R=2491 into the critical range.

During heavy exercise the amount of blood pumped by the heart increases by a factor of three to five. This increases the average blood velocity by the same factor. Critical values for the Reynold's number will be reached even for the blood flow in the main arteries whcih will cause turbulences.



Turbulent flow is less efficient. The flow decreases for the same pressure gradient, the transport mechanism is less efficient and more work is required for the heart.